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A STUDY OF THE CYCLO-ROTATIONAL POWERS OF THE EYES

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ABSTRACT. An expression for the "false torsion" of an eye is derived in terms of the direction of its line of fixation, and is used in calculating the extent of the compensatory cyclo-rotational powers of the eyes.

INTRODUCTION

In a previous paper* a description was given of an investigation carried out to ascertain with some accuracy what movements the eyes execute in performing their normal task of orientating themselves correctly to give two retinal impressions which stimulate those processes necessary to complete our full perception. The investigation was confined to a study of cyclo-movements, i.e. rotations around the front to back axis of the globe, and more particularly to the cases of the fusing of two skew lines. Both objective and subjective experiments were described and results given which clearly showed that two separate processes were at work, the one when the eyes fused skew vertical lines and the other when skew horizontal lines were fused. The results indicated definitely that in fusing skew vertical lines the eyes fuse in succession the small elements of it, running up and down the lines and changing their convergence as they do so; the process being similar in a restricted sense to the case of the eyes viewing a single line on the median plane, leaning away from the observer. If the lines are "fused," the change of convergence of which the eyes are capable, while running over the lines with accommodation fixed (and herein lies the above-mentioned restriction), is sufficient for the purpose. Thus the movements are only those which are required in fusing points—namely medial and lateral rotations—and not rotations of the globes about their front to back axis.

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* J. Adamson, *The Refractionist*, 20, Nov. 1931 (395).

The experiments showed, however, that entirely different processes were brought into play when the eyes were called upon to deal with horizontal skew lines, the results indicating that certain cyclo-rotations can be made in the interests of fusion. The tests carried out on all observers showed that the eyes are capable of greater incyclo-rotation than excyclo-rotation. The experimental values obtained for these rotations were of the order of 7·5 degrees for the incyclo-rotations and 3·5 degrees for the excyclo-rotations.

The eye, by the co-operative action of its three pairs of muscles, is capable of producing such types of rotation as are possible to a body with three-rotational degrees of freedom—within the limits set by the anatomical structure of the whole muscular system. Furthermore, the production of any rotation of the globe requires the harmonized action of all three pairs of muscles, since the axes of the members of each pair are not absolutely coincident. The study of after-images has shown that such co-operative action occurs in rather a definite way, and that the globe rotates about an axis in the vertical plane through the centre of rotation (i.e. Listing's plane) and normal to the plane containing the primary and final directions of the line of fixation. A consequence of such movement is that there enters into each position of the globe a certain amount of "false torsion." When the eye moves from the primary to a tertiary position, the primary vertical section of the eye no longer remains vertical. The image of a vertical line, formed in an eye in a tertiary position, does not, therefore, fall on the retinal meridian which was vertical in the eye's primary position. This change of the vertical meridian of the eye, consequent upon the rolling of the globe about an axis in Listing's plane, is not a rotation of the globe about its front to back axis.

In movements of the eyes in which the lines of fixation remain parallel (versions), the degree of "false torsion" in each eye will be identical and in the same sense. Thus, for parallel movements, corresponding meridians of the two eyes will always remain parallel, and the images of an object will always fall on corresponding points or meridians, no cyclo-rotational movements of the globes being required.

If, however, we consider rotations of the eyes in which the lines of fixation do not remain parallel (vergences), i.e. in near vision the lines of fixation being converged, we can no longer say that each globe is in a similar state of "false torsion," for when viewing near objects the orientations of the globes with reference to the co-ordinate axes will differ from each other. For all states of convergence, therefore, there will be a dissimilarity in the degree of "false torsion" experienced by the two eyes, except for cases in which the near object viewed lies on the horizontal plane through the centres of rotation, since purely lateral rotations of the globe introduce no "false torsion." If both lines of fixation are directed to a point on the median plane, the sense of the "false torsion" will be opposite in each eye. If the point viewed is distant from the median plane by more than half the distance between the centres of rotation of the eyes, the sense of the "false torsion" will be similar in each eye but different in degree.

Thus in all cases of symmetrical convergence with the eyes elevated or depressed the two eyes will have equal and opposite degrees of "false torsion." The images of an object, viewed with the eyes in such positions, would fall on disparate points of the two retinae, and diplopia would exist. Since such diplopia does not make itself manifest, compensating rotations of the globes about their lines of fixation must occur.

In certain parts of the peripheral field of binocular fixation, the inclinations of the two originally vertical meridians, due to "false torsion," should exceed the cyclo-rotational powers of the eyes and produce diplopia. With the right type of object diplopia is obtained. This idea is utilized with a view to obtaining the values for the possible extent of such rotations, under the more normal binocular functioning of the eyes, in viewing a single object, and comparing the results with those obtained by the methods of the previous paper.

For this purpose an expression is required to give the values of the "false torsion" of the eye when the line of fixation is directed to various parts of the field of fixation. To avoid any ambiguity as to the planes of reference, the mathematical processes in the derivation of the expression will be given.

DERIVATION OF AN EXPRESSION FOR THE ANGLE OF FALSE TORSION

Imagine two systems of rectangular coordinates x, y, z and x', y', z' (Fig. 1), the second set being imagined as obtained from the first by a rotation about an axis in the yz-plane (Listing's plane). The set of axes OX, OY, OZ being regarded as fixed in the head, the point O is coincident with the centre of rotation of the globe, and the axis OZ is vertical. The axis OX is the line of fixation of the eye in its primary position. The second set of axes OX', OY', OZ' is fixed in the globe,

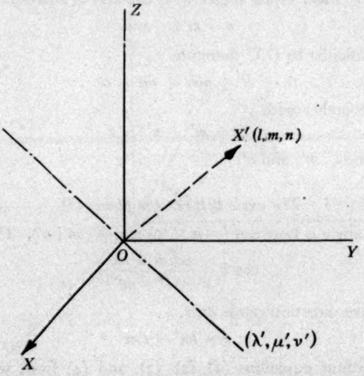

Fig. 1.

and movable with it, OX' being the new position of the line of fixation, and the plane $OX'Z'$ the new position of the primary sagittal section of the globe. The problem is to find the angle between the vertical plane through the new line of fixation OX' and the plane $OX'Z'$.

The vertical plane

Let the new line of fixation OX' have direction cosines (l, m, n), and let the vertical plane through OX' be
$$\lambda x + \mu y = 0.$$
Then
$$l\lambda + m\mu = 0,$$
and the plane is
$$-mx + ly = 0,$$
and the direction cosines of the normal are $(-m, l, 0)$. This gives the direction ratios of the normal which coincides with OY in the initial position.

Axis of rotation

Let the direction cosines of the axis of rotation be (λ', μ', ν'). Then, since it is in the yz-plane,
$$\lambda' = 0,$$
and, since it is perpendicular to OX',
$$\mu' m + \nu' n = 0,$$
and its direction ratios are $(0, n, -m)$.

The plane $OX'Z'$

Let (l', m', n') be the direction cosines of OY', the normal to the plane $OX'Z'$. Then OY and OY' make equal angles with the axis of rotation. Therefore
$$n = m'n - mn' \qquad \text{......(1).}$$
As OY' is perpendicular to OX' therefore
$$ll' + mm' + nn' = 0 \qquad \text{......(2).}$$
These equations together with
$$l'^2 + m'^2 + n'^2 = 1 \qquad \text{......(3)}$$
suffice to determine l', m', and n'.

The angle between the planes (δ)

The required angle is between $(-m, l, 0)$ and (l', m', n'). Therefore
$$\cos\delta = \frac{-l'm + lm'}{\sqrt{l^2 + m^2}},$$
since $(-m, l, 0)$ are direction ratios only.

Let
$$x = lm' - l'm \qquad \text{......(4).}$$

We have now four equations (1), (2), (3), and (4) from which to eliminate l', m', and n' and solve for x in terms of l, m, and n.

Solution of the equations

From (4) and (1) $$l' = \frac{lm' - x}{m}; \quad n' = \frac{(m' - 1)\,n}{m},$$

and, substituting in (2),

$$l\,(lm' - x) + m^2 m' + n^2\,(m' - 1) = 0,$$

and, remembering that $l^2 + m^2 + n^2 = 1$,

$$m' = n^2 + lx.$$

Substituting these values of l', m', and n' in (3) and reducing, we get

$$x^2\,(1 - l^2) - 2l\,n^2 x + n^2 - n^4 - m^2 = 0$$

and $$x = \frac{+\,ln^2 \pm \sqrt{l^2 n^4 + (1 - l^2)\,(m^2 + n^4 - n^2)}}{1 - l^2},$$

which, again using the fact that $l^2 + m^2 + n^2 = 1$, further reduces to

$$x = \frac{ln^2 \pm \sqrt{m^2}}{1 - l^2} \qquad \text{......(5).}$$

Selection of the appropriate solution

In order to decide which solution is appropriate to the problem under consideration, the solutions were tested by applying them to a special case. Now, since x may have its sign changed by simply measuring the other angle between the planes, it is obviously of no value to use a special case in which any one of l, m, or n is zero, since x would then reduce to one term and ambiguity of sign would remain as ambiguity of angle.

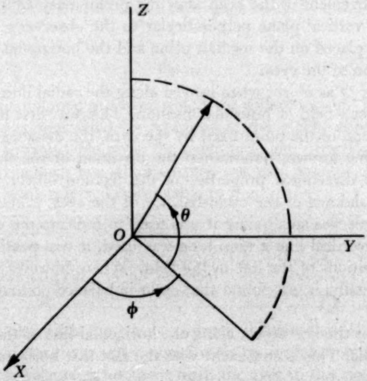

Fig. 2.

The special case selected was a rotation through any angle about an axis in the yz-plane bisecting externally the angle YOZ. The application of this case to the problem shows the appropriate solution to be that given by taking the positive sign of the square root, and the final solution is

$$x=\frac{l^2n+m^2}{1-l^2}=\frac{m^2+ln^2}{m^2+n^2},$$

and
$$\cos\delta=\frac{x}{\sqrt{l^2+m^2}},$$

or, expressing it more conveniently in terms of θ and ϕ, the angles of elevation and declination of the line of fixation OX' (Fig. 2), since

$$l=\cos\theta\cos\phi,$$
$$m=\cos\theta\sin\phi,$$
$$n=\sin\theta,$$
$$\sqrt{l^2+m^2}=\cos\theta,$$

we have
$$x=\frac{\cos^2\theta\sin^2\phi+\cos\theta\cos\phi\sin^2\theta}{\sin^2\theta+\cos^2\theta\sin^2\phi},$$

and
$$\cos\delta=\frac{\cos\theta+\cos\phi}{1+\cos\theta\cos\phi}\qquad\text{......(6)*.}$$

EXPERIMENTAL PROCEDURE

A chart, marked as in Fig. 3 with a set of lines radiating from the point O, was placed in front of the observer. The head was held erect and steady by means of a headrest carrying a horizontal bar which the observer gripped with his teeth to eliminate any movement of the head after the preliminary adjustment. The chart was placed in a vertical plane perpendicular to the observer's median plane and the point O was placed on the median plane and the horizontal plane through the centres of rotation of the eyes.

A small letter O as object when moved along the radial lines sufficed to determine the observer's field of binocular fixation. This was first mapped out on the chart. The position of the point fixed by the eyes, the distance of the chart from the observer being known, determined the direction of the lines of fixation for each eye. Since directional properties of the fixation object were necessary to indicate any breakdown of the cyclo-balance of the eyes, a line object was used; the object perforce was small since it was used to indicate the orientations of the globes. When a vertical line 2 mm. long was used, it was possible to see the line singly for all positions of the line in the field. When, however, the line used was disposed horizontally, it was found that cyclo-imbalance occurred in certain parts of the field.

Movements of the line object along the horizontal line of the chart discovered no sign of diplopia. This is consistent with the fact that horizontal cardinal movements produce no "false torsion," the images formed thus falling on corresponding meridians of the eyes.

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* For cardinal rotations of the globe, i.e. for rotations of the lines of fixation in a vertical or horizontal plane, the angle of "false torsion" $\delta=0$.

On moving the horizontal object along the vertical line of the chart, it was found that the full extent of the field could not be covered before cyclo-imbalance was manifest. Similarly diplopia was experienced before the limit of the field of binocular fixation was reached on moving the horizontal object upward along the oblique radial lines of the chart. No such effect could be recorded in oblique downward movements, since the field is too limited in these directions by facial contours. The positions of the object when cyclo-diplopia showed itself were recorded on the chart.

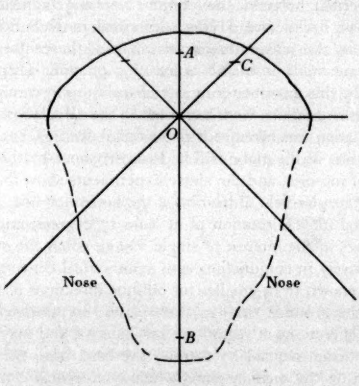

Fig. 3.

A field was thus mapped out over which the differences of "false torsion" in the two eyes were compensated by rotations of the globes about their front to back axes. The following table gives the magnitude and direction of the angle of "false torsion" (δ) in each eye when balance was just possible, the directions of the line of regard being referred in the table to Fig. 3.

Direction of regard	Right eye	Left eye	Total
(1) Vertically, upward (A)	4° 1′ (incyclo)	4° 1′ (incyclo)	8° 2′ (incyclo)
(2) Vertically, downward (B)	8° 36′ (excyclo)	8° 36′ (excyclo)	17° 12′ (excyclo)
(3) Obliquely, upward (C)	3° 3′ (excyclo)	5° 47′ (incyclo)	2° 44′ (incyclo)

It will be seen that with the eyes converged and elevated, if no compensating cyclo-rotation took place, the eyes would each be incyclo-rotated by 4 degrees. Thus, while muscle balance is operative with the eyes in this position, a compensatory excyclo-rotation of 4 degrees must be present in each eye. With the eyes converged and depressed, the limiting position of the muscle balance necessitates a compensatory incyclo-rotation of 8·5 degrees in each eye. In the oblique direction the "false torsions" are in the same sense and a compensatory excyclo-rotation of only 1·3 degrees in each eye is required to bring the meridians of the globes into correspondence.

In calculating the values of the angle δ a position for the centre of rotation of the eye had to be assumed. George, Toren and Lowell* found that the centre of rotation of the globe can be regarded as a fixed point for a range of rotations about a vertical axis between 30 degrees nasalward and 20 degrees temporalward, situated 15·4 mm. from the corneal vertex and 1·65 mm. nasalward from the visual axis. In the experiments cited, however, the rotations were not confined to the horizontal. The experiments of Berlin and Müller, described in Helmholtz's *Physiological Optics*†, showed that the maximum displacement of the centre of rotation when vertical rotations are made is of the order of 0·47 mm. Displacements of the centre of rotation by this amount result in a change of only 2 minutes in the value of the angle δ. Such variations were neglected in the calculations and the position of the centre of rotation was taken as that quoted above.

Any cyclo-rotation of the globe will be brought about by the combined action of all three pairs of muscles, and the above experiments show that their combined actions, over the binocular field of fixation of the horizontal line object, are capable of producing a total incyclo-rotation of at least 17 degrees, and a total excyclo-rotation of 8 degrees in the interest of single vision, when the eyes are depressed or elevated respectively in conjunction with symmetrical convergence. The cyclo-rotational range is shown to be smaller for oblique directions of regard. This may be explained on the grounds that, in these cases, asymmetrical convergence is required and such directions of regard are not habitual and may be considered as not truly normal, being avoided by turning the head until the point of fixation assumes a position on the median plane when symmetrical convergence is again established. It must be remembered that, in each position of breakdown of the muscle balance, the direction of regard is an unnatural one, and that the cyclo-rotations calculated above will be far in excess of those ever required in normal binocular vision.

CONCLUSIONS

The above results are taken to indicate that, within the field of complete muscle balance, cyclo-rotations of the extent stated were operative, and not that the eyes reached a certain degree of cyclo-rotation beyond which they were incapable of making further cyclo-rotations. This is proved by the directions of tilt of the mentally projected images of the line object when diplopia occurs. When the eyes reach the condition of cyclo-imbalance, crossed diplopia occurs. When imbalance is manifest with elevation of the plane of regard, the directions of the projected images are as shown in Fig. 4 *a*, and with depression of the plane of regard as in

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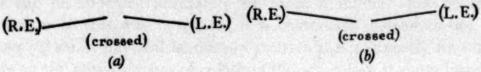
* George, Toren and Lowell, *Amer. Journ. Opth.* **6**, Oct. 1923 (833).

† Helmholtz, *Physiological Optics* (English translation), **3**, p. 135.

Fig. 4 *b*. The projected images to the left are those of the right eye and the ones to the right those of the left eye. This shows that in the upper part of the field of fixation, where excyclo-compensations are necessary, when diplopia occurs the condition is one of excyclo-rotation combined with divergence. In the lower field, where incyclo-rotations are required, when diplopia occurs the eyes are in the condition of incyclo-rotation combined with divergence. When the direction of regard just exceeds its upper limit in an oblique direction, the state is one of excyclo-rotation and divergence and also a vertical imbalance. This clearly shows that the imbalances are not those obtained by a breakdown of the compensatory cyclo-rotations because they reach their limiting values.

The cyclo-rotational powers of the eyes, dependent as they are on the co-operative action of all three pairs of muscles, must require a definite distribution of energy between all the muscles. In the cases of cyclo-imbalance under consideration the eyes are beginning to experience difficulty of convergence and elevation or depression, the energy supplied for this purpose being partly used in doing work against the forces resisting the globe's rotation. Consequently, to produce the requisite lateral and vertical rotations in the peripheral parts of the field, more energy is required. It seems, therefore, that this is the case and the supply of such additional energy entails also an excess of energy being supplied to the oblique muscles, and cyclo-imbalance is caused by excessive action of the obliques and not their deficient action. This seems a reasonable assumption to make since there are numerous examples of muscle imbalance caused by a certain energy distribution making one set of muscles "over-step" the mark—as for example those cases of ocular muscle imbalance of accommodative origin. This would account for the directions of rotation of the projected images, when diplopia occurs, being opposite to what would be expected if it were simply a case of breakdown of those rotations correcting the "false torsions" of the eyes.



(*a*) (*b*)

Fig. 4.

It was found in the experiments using two objects, one before each eye, that the measurements of the cyclo-rotational powers of the eyes were quite valueless if the objects were not perfectly level—a fact which stresses the part played by the vertical rotators of the eyes in upsetting cyclo-balance. This fact is also borne out by clinical experience of muscle imbalance, great care having to be taken to adjust for vertical imbalances before re-educational exercises of the muscles are attempted.

When a single object on the median plane is viewed, the vertical and lateral rotations are symmetrical, and the difficulties of convergence and elevation or depression, experienced in the peripheral fields, will also be symmetrically distributed. In oblique directions of regard, however, such difficulties will no longer be distributed symmetrically between the two eyes and we should expect the cyclo-balance to break down sooner owing to the introduction of a vertical imbalance due to asymmetric actions of the vertical rotators. This explains the smaller cyclo-rotational powers of the eyes for oblique directions of regard, particularly since a vertical imbalance is manifest when the cyclo-balance breaks down for such directions of regard.

Under normal conditions of binocular vision the eyes will rarely, if ever, be called upon to use compensatory cyclo-rotational powers exceeding 1·6 degrees. Since these investigations show them to be capable of compensatory cyclo-rotations far in excess of this value, cyclo-imbalance should rarely be met with unless it is associated with other imbalances—particularly vertical imbalances. The excessive cyclo-rotating powers of the eyes, over that normally required, accounts for the infrequency of cyclo-imbalance and suggests that such imbalances are (unless of anatomical origin) due to vertical and lateral faults. This is again borne out by clinical experience, it being found that such imbalances automatically disappear with a re-education of the other imbalances.

A NEW TRICHROMATIC COLORIMETER

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MS. received, 7th April, 1932. Read and discussed, 9th June, 1932.

ABSTRACT. The design and construction of the instrument are described. It is on a new principle, the intensity of the comparison field being altered by an iris diaphragm, and the colour by moving a magenta-yellow and a blue-magenta filter relatively to one another. Results are given which have been obtained with the instrument.

It has been well known for more than seventy years that all colours can be produced by superposing three coloured lights on a screen, and that consequently any colour can be specified accurately by three numbers. Yet no method of specifying colour in this way is in use; it is customary instead to use names or colour atlases. Also in physical laboratory courses it is extremely unusual to find experiments on the measurement of colour, although colour is more important and interesting to the average student than the properties of the spectrum. One reason for this is that there is on the market no simple instrument for the measurement of colour which is based on the theory of colour. The Guild colorimeter is too expensive to come into wide use. J. Guild* states that "there is practically an open field for the design of really precise instruments on a sound theoretical basis for performing all or any of the processes of colorimetry." L. C. Martin and W. Gamble† state that there is a great need for an apparatus for measuring colour which should be simple, inexpensive, and easily made all over the world.

Consequently, when I was asked in 1928 to communicate a paper to the Colour Photography Section of the International Congress of Photography, I devised a new colorimeter which is described on p. 431 of the *Proceedings* of that congress. Mr Guild and Prof. Luther made criticisms during the discussion of this paper, and I was not satisfied with the instrument myself. Some seven or eight members spoke to me subsequently during the congress on the subject of colorimetry. From their remarks I perceived there was a strong desire for a simple trichromatic colorimeter, and in the autumn of the same year I began a new and more thorough attempt to devise such an instrument. This paper describes the result of that attempt. Owing to pressure of other work it is only now that the instrument has been completed and tested.

In devising a trichromatic colorimeter we have to decide whether the instrument is to work by electric light or daylight, what primaries are to be employed, and how they are to be mixed. Daylight is so inconstant in its quality that I decided to use electric light, but make provision for the occasional use of daylight for special purposes. As primaries I decided to use the Wratten tricolour filters or their equivalents; they divide the spectrum into three parts, and, though not monochromatic, have the advantages of being well known and bright. In mixing the primaries, the fact that the instrument is to be simple rules out the employment of rotating prisms or lenses, such as in the instruments of Guild and Ives. Experiments on diffusing spheres, boxes, and mirror-lined corridors convinced me that the advantage lay with the mirror-lined corridor. But for the advantage of this method to be exploited in full it is necessary for the coloured sources to touch one another. Three slits at the end of a corridor do not mix well, because the opaque spaces between them constitute an initial handicap.

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* J. Guild, *Proc. Opt. Conv.* 1926 (132).

† L. C. Martin and W. Gamble, *Colour and Methods of Colour Reproduction*, 1923 (29).

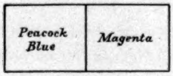

Fig. 1.

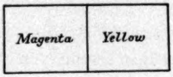

Fig. 2.

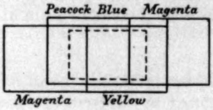

Fig. 3.

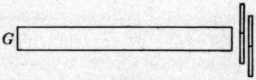

Fig. 4.

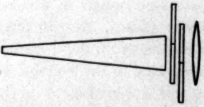

Fig. 5.

I therefore employed the following device. The Wratten tricolour complementary filters were used, a magenta and a peacock blue rectangle cemented together between glass plates as in Fig. 1, and a magenta and a yellow rectangle cemented together between glass plates as in Fig. 2. These filters were then superposed as in Fig. 3, and viewed through a rectangular aperture indicated by the broken lines. The central area of the aperture is illuminated by light which comes through yellow and peacock blue and is consequently green. Similarly the area to the right is red and the area to the left blue. The proportions covered by red, green, and blue can be altered by sliding the filters relatively to one another and to the aperture through which they are viewed.

The filters were at first employed in conjunction with a mirror corridor 10 inches long and of square section, as indicated in Fig. 4, the side of the square being 1 inch. At the end is a piece of ground glass *G*. The beam entering through the filters was strongly divergent and multiple reflection took place at the sides of the corridor, forming images of the filters. Thus the rays from each colour fell on the ground glass at several angles and an observer viewing the ground glass saw it of a uniform colour, i.e. it did vary in colour with the direction of the eye, as it would have done had there been no reflection at the sides of the corridor.

The mirror-lined corridor was afterwards replaced by a wedge of very clear glass, a plan of which is given in Fig. 5. This was 5 inches long along the axis, ½ an inch high, 1 inch broad at the wide end, and ¼ inch broad at the narrow end. The sides, top, and bottom were polished, and the ends were ground matt. Light diverged from an iris diaphragm, was made parallel by the lens shown, passed through the filters, and fell on the matt end of the wedge which made it diverge. The rays, with the exception of the most divergent ones, were kept inside the wedge by total reflection and ultimately arrived at the other end. This arrangement mixed the colours better than the mirror-lined corridor, and the surface for receiving the rays was better defined and could be placed with greater accuracy than in the case of the corridor.

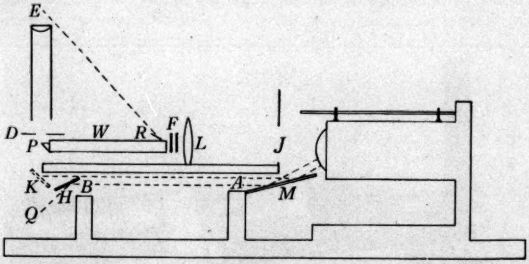

Fig. 6.

Fig. 6 is an elevation of the instrument. The base is 22 inches long. The lamp is a 220-volt, 40-watt gas-filled one with a matt surface, and it was run off the 250-volt lighting circuit. *J* is the iris diaphragm which has a maximum diameter of 1⅜ inches. It is placed at the focus of the lens *L* which has a focal length of 3⅞ inches. *F* is the filters and *W* the wedge. The rays from the matt end of the wedge are totally reflected upwards by the prism *P* to reach the eye of the observer which is situated at *E*. At the upper end of the tube is a lens of focal length about 6 inches, so that the surface of the prism is viewed under a weak magnification. *D* is a diaphragm with a circular hole which restricts the diameter of the field to 2°. *R* is a reflector; by raising his eye from the tube the observer sees in it the upper half of the second filter and can tell at a glance the proportions of the three colours in the mixture. The filters are moved by rack and pinion and their positions read on a horizontal scale by means of two pointers.

So much for the comparison field. The coloured object to be investigated is placed at *Q* and illuminated by rays which fall on the mirror *M*, pass through holes *A* and *B* in the wooden supports, and are reflected by the mirror *H* so as to fall on the object at an angle of 45° with the normal. If the colour is too saturated, it can be desaturated by placing a transparent glass plate at *K* so as to catch the rays which pass over the top of the mirror *B*. If the colour of a solution is to be investigated, the glass trough containing it is placed between *A* and *B*. The intensity of the beams reflected by *M* has been regulated by stops, wire gauze screens, and fogged plates; the colour to be tested is supported on a block, and the mirror at *K* by a clamp. These, as well as a lens which is sometimes used to blur the texture of a coloured cloth, are not shown in the diagram.

The half tone (Fig. 7) shows the appearance of the instrument except that the mirror *H* has been accidentally tilted from the proper angle, and that a cardboard cover has been removed from the wedge. The viewing tube is detached when the instrument is fitted into its case.

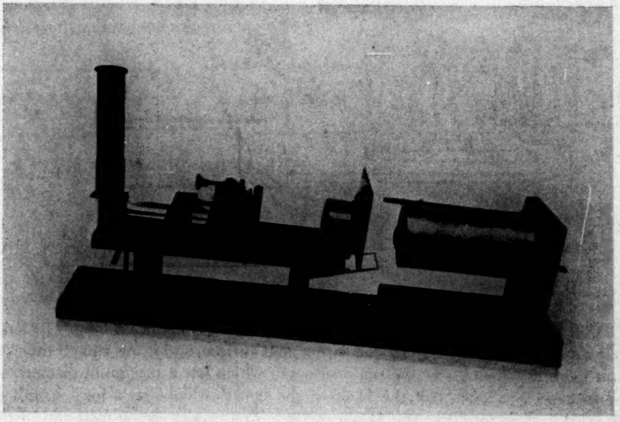

Fig. 7.

The two pieces of gelatine in each filter are not in perfect contact, there being a separation of the edges in the one case of $\frac{1}{10}$ mm. and in the other case of $\frac{1}{30}$ mm. This is not enough to cause appreciable error. I am informed that the two halves are placed in contact, but that they draw apart during the six weeks the cement is left to dry.

There is one scale, and as each filter is displaced a pointer moves along this scale. It is important to determine the positions of the pointer corresponding to no red and no blue and the relative position corresponding to no green. This can be done by removing the reflector and sighting the filters over the top of the glass wedge. The position for no red can also be determined by the change in the colour of the comparison field, as red is screwed over the edge, or the change in luminosity when the comparison field is viewed through a red filter and red is screwed over the edge, but the position for no blue cannot be determined accurately by these methods owing to the feebler luminosity of that colour.

The comparison field is not perfectly uniform for all positions of the filters. It is frequently redder on the right and bluer or greener on the left. This difference is not perceptible at first on looking into the tube, but gradually becomes visible. It is not a source of error, if balance is made always in the middle of the field, and it is a help in obtaining balance, for the two sides of the field are wrong in different directions. It can be removed, if necessary, by substituting a vernier microscope for the tube.

When the colours are mixed, as in the Guild colorimeter, by a rotating prism and persistence of vision, we can take it for granted that the quantities of the colours in the mixture are proportional to the areas of the slits. We cannot make the equivalent assumption here, because, though the quantities of the colours going into the wedge are proportional to the areas in front of the end, it is not certain they will be scattered in any given direction in the same proportions. To investigate this point one of the filters was covered with black paper, leaving a magenta strip of 3 mm. width clear. This was combined with the yellow of the other filter, so as to make a red strip 3 mm. wide which could be moved across the end of the wedge. As test object a piece of cardboard was set up at 45° to the vertical and illuminated with light of the same colour by a lamp which could be moved along a photometer bench, and thus the brightness of the comparison field was measured for different positions of the strip. It was not always the same, the first and last positions not having the same illuminating power as the others. The discrepancy varied with the setting of the iris diaphragm. From the results it was possible to compare the actual illumination given by an area with the magnitude of the area itself, and Fig. 8 gives the result for one setting of the iris diaphragm. If the illumination were strictly proportional to the area, the points would lie on the straight line. I do not know yet whether the deviations can be ignored. They can be allowed for by calibrating the scale for different settings of the iris diaphragm, and could be con-

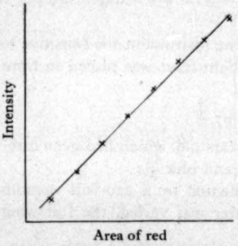

Fig. 8.

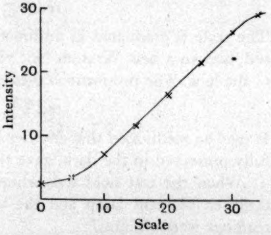

Fig. 9.

siderably reduced by increasing the length of the wedge in comparison with its breadth.

Fig. 9 gives the calibration curve of the iris diaphragm. The abscissae give scale readings proportional to the angle through which the rim was turned. The ordinates give in arbitrary units the intensity transmitted.

In order to measure the maximum brightness of the comparison field, a piece of white cardboard set at 45° to the vertical was, as before, used as test field, and illuminated by a lamp which could be moved along a photometer bench. The iris diaphragm was set at its widest, and the comparison field made in succession wholly red, wholly green, and wholly blue. The lamp was then adjusted so that the white test field was as bright as the coloured comparison field, and it was found that the red, green, and blue were balanced respectively by illuminations of 336·0, 81·4 and 4·9 metre-candles on the cardboard.

When a piece of white paper was used as test object, the proportions necessary to match it were

red 3·3, green 12·6, blue 9·5.

The scale is graduated in millimetres. This makes the instrument too sensitive to red, and so a new Wratten No. 78 tungsten-to-daylight filter was placed in front of the lens. The proportions then were

red 6·7, green 12·0, blue 6·7.

It may be mentioned that another No. 78 filter ten years old, which had been carefully preserved in the dark, gave red 5·5, green 14·8 and blue 5·1.

When the test field was white cardboard illuminated by a 220-volt vacuum tungsten filament lamp and the voltage on the latter was varied, the following readings were obtained:

Volts	Diaphragm scale	Red	Green	Blue
250	35	6·1	14·5	4·8
200	13	7·0	13·6	4·8
150	5	7·9	12·7	4·8

Powdered potassium dichromate gave iris diaphragm 22, red 13·3, green 10·0, blue 2·1. Under the same conditions powdered copper sulphate gave iris diaphragm 31, red 5·1, green 15·6, blue 4·7.

In order to test the instrument some of the colours in Ridgway's *Colour Standards and Nomenclature* were measured. This book is an atlas of 1115 colours, with descriptive matter arranged systematically and based upon 36 full colours. Every third of these was near enough the edge of the page to pass under the instrument, and Table I gives the result of its measurement. The first column gives the name of the colour, the second a number proportional to the intensity, not the scale reading of the diaphragm, and the third and fourth the quantities of red and blue in the colour. The quantity of green can be obtained by subtracting the sum of the third and fourth from 25·4.

Table I.

Colour	Intensity	Red	Blue
1 Scarlet	5·3	14·0	4·0
2 Orange chrome	9·0	11·8	3·9
3 Chrome yellow	11·8	9·9	3·6
4 Lemon yellow	15·2	7·9	3·5
5 Neva green	10·8	6·1	3·2
6 Emerald green	7·0	4·5	3·2
7 Benzol green	4·2	2·4	9·8
8 Methyl blue	3·5	3·2	10·0
9 Phenyl blue	2·7	5·2	10·0
10 Spectrum violet	2·7	7·9	10·0
11* Purple (true)	3·4	12·8	9·5
12 Rose red	4·6	14·9	4·4

The positions of the colours are plotted in Fig. 10. None of them is saturated enough to require negative coefficients.

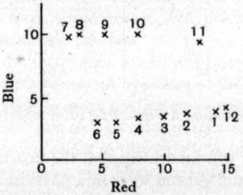

Fig. 10.

Table II gives measurements made on the vertical row of colours next the edge of the page on Plate 1 in Ridgway's book. This gives one hue, scarlet, with varying proportions of white and black added.

Table II.

Colour	Intensity	Red	Blue
White	20·5	7·6	3·0
Shrimp pink	14·5	9·4	3·0
Strawberry pink	11·5	10·8	3·0
Peach red	11·0	12·0	3·0
Scarlet	5·3	14·0	4·0
Brazil red	4·6	14·3	3·0
Morocco red	2·7	13·8	3·0
Claret brown	2·7	11·2	3·0

As a final test the instrument was used to measure the colour of the four brightest lines in the helium spectrum. A piece of ground glass, matt side down, was laid on the upper surface of the prism *P*, so that it projected over the edge, the eyepiece removed from a spectroscope which had a large dispersion, and the rays reflected upwards so as to form an image of the line in question on the ground glass close to the edge. The four lines were too saturated to be matched directly. It was necessary to superpose a desaturating light. This was done by means of the thin glass plate K indicated in Fig. 6. If r, g, and b are the areas matching the sum of the two lights, r', g', b' the areas matching the desaturating light alone, and I and I' the intensities corresponding to the two apertures in question, the areas matching the spectral line alone are proportional to

$$r - I'r'/I, \quad g - I'g'/I, \quad b - I'b'/I,$$

and to get their absolute values it is necessary only to multiply these quantities by the factor that will make their sum 25·4. The results are given in Table III; they do not represent the highest accuracy obtainable with the instrument, but are such as might be obtained in an hour or two.

Table III.

Wave-length Å	Red	Green	Blue
4471	0	− 2·3	27·7
5016	− 3·1	19·9	8·6
5876	11·6	17·3	− 3·5
6678	122	− 64	− 83

An endeavour will be made to find a correcting filter which will give pure red smaller values.

From the foregoing it will be evident that the instrument is admirably suited for educational purposes and general work in a physical laboratory. It is too soon yet to estimate its value for commercial purposes or for research of the highest accuracy. Messrs Adam Hilger Ltd. are considering the addition of this instrument in a modified form to the list of instruments which they manufacture.

DISCUSSION

Mr J. Guild: Dr Houstoun can always be relied upon to introduce a note of originality whether he is designing instruments or constructing visual theories. With the utility of the present apparatus as a medium for teaching purposes I need not concern myself. It all depends on what a student is desired to learn as to what kind of apparatus is necessary to demonstrate it. The university students who pass through Dr Houstoun's hands are probably not specially interested in the technical problems of colorimetry.

As regards the possible utility of the instrument in meeting the needs of technical workers in the laboratories of the colour industries it is necessary to point out to anyone interested in the design of such instruments that the demand which exists for a cheap and simple colorimeter is not a demand for something of a lower standard of performance than that of existing types. We have become fairly familiar in recent years at the National Physical Laboratory with the requirements, real or imagined, of those who are endeavouring to apply colorimetry to industrial problems, and some of these requirements demand a standard of performance higher than that of any available colorimeter, and higher, probably, than will ever be reached. There is no demand whatever for any instrument which is not capable of equalling or exceeding the performance of the Guild colorimeter. It is this consideration which has prevented us from developing cheaper instruments ourselves long ago. We have found it easy to evolve many comparatively simple colorimeters which were not quite so good as the present model, but have not yet found it possible to construct one which is simpler and at the same time as good or better.

It must be borne in mind that the criterion of cheapness must not be applied to initial cost only but also to cost of operation. In Dr Houstoun's instrument, for example, there are several features which would increase cost of operation. His particular method of colour-mixing limits his choice of primaries to the Wratten tricolour filters. The "working triangle" provided by these primaries excludes quite a fair proportion of the colours of industry. The use of the desaturation process would be called for much more frequently than is the case with the Guild colorimeter. Theoretically this is no disadvantage, but practically it involves an extra process and therefore consumes extra time.

Then it appears that the instrument scales must be corrected from a series of calibration curves appropriate to various openings of the intensity diaphragm. The application of such corrections to every reading consumes time which, though small in each case, soon becomes serious. In a laboratory where a colorimeter was in regular daily use, the aggregate effect of various slight additional inconveniences of this kind would very soon consume the difference in first cost between a very cheap and a very expensive instrument.

However, I do not wish to suggest at the present stage that Dr Houstoun's colorimeter may not prove useful. I understand that he intends to explore its possibilities, from the technical point of view, in the next few months. He may find it possible to remove the defects to which I have alluded without materially increasing the complexity of the design. If this proves to be the case and if the accuracy obtainable in actual numerical results is of the order required in technical work, the instrument will be a welcome addition to available colorimetric equipment.

Although it does not directly arise from Dr Houstoun's paper, I may take the opportunity of suggesting that the problem of facilitating the widespread use of colorimeters by all those who really need them calls for some investigation, on the part of instrument makers, of methods of cheapening production of existing types as well as for the development of new types. The original trichromatic colorimeter made at the National Physical Laboratory was *not* an expensive instrument. In its essentials the design of this colorimeter is not complex. As a matter of fact it is the simplest possible design which will adequately perform the necessary functions. The inclusion of a rotating element is regarded by some as objectionable, but rotating elements are regularly used in all kinds of photometric work and are not regarded either as unnecessarily complex or as an expensive luxury. The rotating prism of the Guild colorimeter solves the mixing problem in the simplest possible way with the minimum loss of light. The only justification for even troubling to look for an alternative method lies in the purely adventitious fact that the widespread replacement of direct current by alternating current in the public supplies through the country introduces difficulties arising from superposed flicker in laboratories where a direct current supply is not available.

In the meantime, may I suggest with all humility to my friends of Adam Hilger, Ltd., that while continuing to encourage the development of new types of colorimeter they should also investigate the possibility of incorporating the really very simple optical system of the Guild colorimeter in a much cheaper setting?

Dr Houstoun: I am in general agreement with Mr Guild's remarks. I would like only to make two points in reply:

(i) The filters are not limited to the Wratten complementaries; narrower strips of the spectrum may be used with, of course, some loss of light.

(ii) My aim has been to make an instrument for teaching purposes. For this a very accurate colorimeter is not required. To quote an analogy, we can do quite a lot with a one-prism spectroscope. But when developed and standardised the instrument *may* prove equal to the highest requirements. I have not yet seen anything to suggest that this is impossible.

THE PARALLEL PLATE MICROMETER

By Instructor-Captain T. Y. BAKER, F.Inst.P., R.N. (Ret.)

MS. received, 30th March, 1932. Read and discussed, 9th June, 1932.

ABSTRACT. A plate of parallel glass placed in the convergent beam of a reading microscope is a useful means of obtaining the "fine-reading" of a divided circle. The fine-reading scale is uniform if its length is proportional to the tangent of the angle through which the plate is tilted and if the refractive index of the glass is 1·60.

The "fine-reading" of a divided circle is often made by means of a micrometer microscope. An image of the scale divisions is formed in the microscope and a moveable cross-hair adjusted until it lies on a division mark. The amount of travel of the line from its zero position is measured by a micrometer screw, the fine reading being taken direct from the divided micrometer head. An alternative method is to use a fixed cross-hair in the microscope and move the image of the division marks by some optical means without altering the setting of the circle. Perhaps the best known optical device of this character is the travelling prism of small deflecting angle which is made to move along the axis of the microscope.

Another method which, for some reason or other, does not seem to have been extensively employed is the use, in the convergent beam of the microscope, of a parallel plate of glass which, when tilted about an axis parallel to the plate, causes a small lateral displacement of the image proportional to the thickness and to the amount of the tilt.

When a ray of light passes through a parallel plate of glass at an angle α with the normal it emerges parallel to its original direction but laterally displaced by an amount $t\alpha(1 - 1/n)$, to a first approximation, t being the thickness of the plate, α the tilt (in circular measure) of the plate from the normal, and n the refractive index.

The exact value of the displacement is $t(\sin\alpha - \tan\delta\cos\alpha)$ where $\sin\alpha = n\sin\delta$, or δ is the angle with the normal in the glass. There is no approximation at all in this formula. The displacement of the image in the field of view is thus the product of the thickness with a factor $\sin\alpha - \tan\delta\cos\alpha$ which depends only on the tilt and the refractive index.

For convenience of the user it is advantageous to have the amount of the fine reading visible in the field of the microscope and this is conveniently done by making the tilt of the plate move a scale which is in focus in the field of view and can be read against a fixed pointer. The simplest method of all is illustrated diagrammatically in Fig. 1. A circular scale cut on a transparent material is attached to the parallel plate and can be read against a pointer but owing to the nature of the factor $\sin\alpha - \tan\delta\cos\alpha$ the scale is not quite uniform. In Fig. 2 a sliding scale is actuated by an arm attached to the plate, a pin on the scale travelling in a slot in the arm. The displacement of the scale is thus proportional to tan α. In the third case (Fig. 3), which is similar to the second, a pin on the arm travels in a slot in the scale and the displacement of the scale is proportional to sin α. The degree of uniformity or otherwise of the scales in these three cases is examined below. It is evident that the best method to use is that one which gives the nearest approach to uniformity so that, when such a scale is used, the error will be least. To test the point, the factor $\sin\alpha - \tan\delta\cos\alpha$ must be plotted against α, $\tan\alpha$, and $\sin\alpha$.

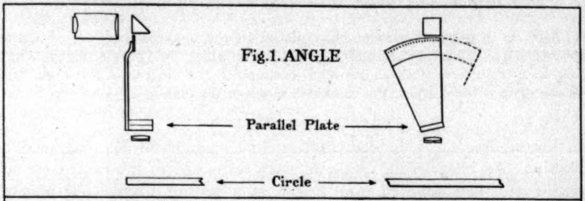

Fig.1. ANGLE

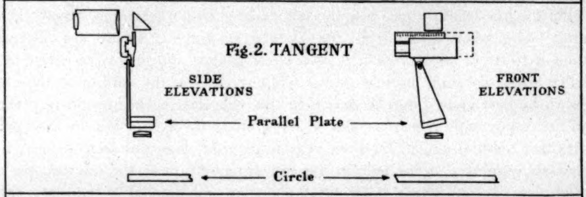

Fig.2. TANGENT

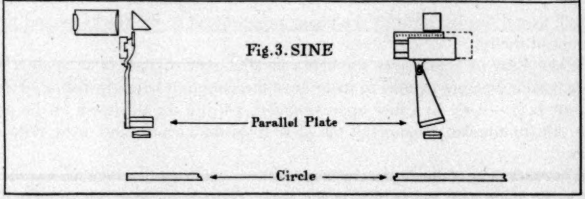

Fig.3. SINE

If the displacement factor is expanded we obtain the following values:

(1) $(1 - 1/n)\{\alpha - \alpha^3(n^2 - 3n - 3)/6n^2 + \alpha^5(n^4 - 15n^3 - 15n^2 + 45n + 45)/120n^4\}$,

(2) $(1 - 1/n)\{\tan\alpha - \tan^3\alpha\,(n^2 - n - 1)/2n^2 + \tan^5\alpha\,(3n^4 - 5n^3 - 5n^2 + 3n + 3)/8n^4\}$,

(3) $(1 - 1/n)\{\sin\alpha + \sin^3\alpha\,(n + 1)/2n^2 + \sin^5\alpha\,(n + 1)(n^2 + 3)/8n^4\}$.

In (1) the coefficient of the cubic term is positive and that of the fifth power term is positive for all possible values of the refractive index and it is evident that there can be no hope of making the scale uniform except for small values of the tilt. Exactly the same can be said of the sine expansion (3). But with the tangent expansion (2) it is a different question. The coefficient of the cubic term becomes zero for $n = 1{\cdot}618$ and although the coefficient of the fifth power term does not vanish for this value of n yet it is not large. If the value of n be taken as 1·600 then the cubic and fifth power terms are of opposite signs and tend to neutralise one another. The calculated values of the function for values of $\tan\alpha$ in uniform steps of 0·01 up to 0·30 are given in Table I and alongside are the errors of a uniform scale which has the correct values at $\tan\alpha = 0$ and $\tan\alpha = 0{\cdot}30$.

Table I.

$\tan\alpha$	True value	Error	$\tan\alpha$	True value	Error
0·00	0	0	0·16	0·060006	0·000011
0·01	0·003750	0	0·17	0·063757	0·000013
0·02	0·007500	0·000001	0·18	0·067508	0·000014
0·03	0·011250	0·000001	0·19	0·071260	0·000016
0·04	0·015000	0·000002	0·20	0·075012	0·000018
0·05	0·018750	0·000002	0·21	0·078763	0·000019
0·06	0·022500	0·000002	0·22	0·082514	0·000020
0·07	0·026251	0·000003	0·23	0·086262	0·000019
0·08	0·030002	0·000004	0·24	0·090008	0·000016
0·09	0·033752	0·000005	0·25	0·093756	0·000014
0·10	0·037503	0·000006	0·26	0·097504	0·000012
0·11	0·041257	0·000006	0·27	0·101252	0·000010
0·12	0·045004	0·000008	0·28	0·105000	0·000009
0·13	0·048755	0·000009	0·29	0·108745	0·000004
0·14	0·052506	0·000011	0·30	0·112491	0
0·15	0·056256	0·000011			

The extreme tilt is rather less than 17° and, clearly, the plate can be tilted from − 17° to + 17° without introducing any greater error than 0·000020. The meaning of this can be appreciated by considering the case of a circle cut to 20′ which has to be read to the nearest second on the micrometer. The full range of movement of the scale is from − 0·112491 to 0·112491 so that 20′ corresponds to 0·224982 and the maximum error to just more than one tenth of a second.

When the expansion is made in terms of the angle formula (1), the errors are as shown in Table II.

Table II. Error.

α	$n = 1{\cdot}5$	$n = 1{\cdot}6$	$n = 1{\cdot}7$
0·00	0	0	0
0·05	0·000583	0·000560	0·000551
0·10	0·001086	0·001028	0·001007
0·15	0·001344	0·001461	0·001243
0·20	0·001320	0·001530	0·001217
0·25	0·000941	0·000895	0·000850
0·30	0	0	0

For an expansion in terms of the sine formula (3), the errors are even larger, as shown in Table III.

Table III. Error.

$\sin\alpha$	$n = 1{\cdot}5$	$n = 1{\cdot}6$	$n = 1{\cdot}7$
0·00	0	0	0
0·05	0·000856	0·000878	0·000882
0·10	0·001573	0·001612	0·001621
0·15	0·002005	0·002055	0·002065
0·20	0·002002	0·002046	0·002058
0·25	0·001395	0·001425	0·001430
0·30	0	0	0

It is clear from an examination of these tables of errors that the tangent formula, when the refractive index is 1·6, is a long way superior to either of the others, though the tilt of the plate through such a large angle may produce astigmatic defects of image which are too serious to be borne.

For a parallel plate the axial displacement of the primary focus is

$$t(1 - 1/n)(n^2 - 3n - 3)\tan^2\alpha/2n^2$$

and that of the secondary focus

$$t(1 - 1/n)(n^2 - n - 1)\tan^2\alpha/2n^2.$$

The decision as to whether the astigmatism can be tolerated depends partly on the thickness of plate that has to be used. As an illustration we may consider the case of the fine-reading of a theodolite referred to previously. With a circle of diameter $3\frac{1}{2}$ inches the 20′ division is about 0·01 inch wide. This might be magnified three times, making the image width 0·03 inch. If we use the parallel plate to the full extent previously considered the total displacement of an image division is $0{\cdot}225t$. Thus the requisite thickness of the plate is 0·133 inch. The primary focus is displaced 0·0046 inch for the extreme tilt and the secondary 0·00004 inch. If the aperture and focal length of the eye-piece of the micrometer give sufficient depth of focus the plate can be used to the extreme tilt; if not, the tilt must be diminished and the fine-reading scale made less open. In any case it seems to be evident that the tangent expansion with a glass of refractive index 1·6 is the best scheme to adopt if the scale is to be uniform.

Although the astigmatism defect is not too serious, the same cannot be said for defects of colour. For a change of wave-length from C to F the displacement of the image is altered by approximately 1 per cent. This is slightly compensated for by the second order term when the tilt of the plate is greatest but not enough to have any appreciable effect on the error. The scale will be 1 per cent. in error if it is graduated for one colour and another is used. If we take the case of subdividing 20′ on the scale in order to read to single seconds, the total scale will be 12″ longer in one case than in the other. One will not make an error of this magnitude in the reading. One will be endeavouring to fit in between the parallel datum lines an image which is coloured red on one edge and blue on the other. It is reasonable to suppose that one will be able to strike an approximate mean but even then one must anticipate errors of reading of a couple of seconds or so if white light is used to illuminate the scale.

This difficulty can be overcome in the laboratory by the use of a monochromatic illuminator. In the field by the use of a filter restricting the light to a narrow band of wave-lengths it can be less completely overcome. The device would be quite satisfactory in the field if only someone would invent a portable source of monochromatic illuminant that can be run off a low voltage accumulator.

DISCUSSION

Mr T. Smith: The theoretical advantages of the tangent calibration to which Capt. Baker has directed attention depend more upon the fact that the expansions in α and in $\sin\alpha$ have all their terms of the same sign than upon the possibility of making the term in $\tan^3\alpha$ vanish. If their signs in the other two expressions alternated, the one departure could be satisfactorily offset by another.

The chief difficulty in the use of this micrometer being due to colour differences, it seems natural to abandon the restriction to a simple parallel plate and to consider whether the recognised defects can be eliminated by a somewhat more complex structure in which spherical surfaces are used. As far as I know this problem has not been investigated, and it is not apparent at the outset that an advantageous solution does not exist. A form with freedom from colour, astigmatism, and change of focus, with adjustment by simple rotation and a scale preferably uniform in angle, would be so marked an improvement that a comparatively complex construction would be admissible.

Mr J. Guild: Would it be impossible to obtain a sufficient approach to monochromatism consistent with sufficient illumination under field conditions by means of one of the various narrow-band filters which are available?

Capt. Baker: Mr Smith is only right in his statement about the signs of the successive terms of the three expansions if he restricts the values of n to those which are practically possible as refractive indices. As a piece of algebra in which all possible values of n are considered the graph of $\sin\alpha - \tan\delta\cos\alpha$ will change shape with variation of n. It will be most nearly represented by $(1 - 1/n)\,\alpha$ when n is approximately either $-0{\cdot}8$ or $3{\cdot}8$; by $(1 - 1/n)\tan\alpha$ when n is approximately either $-0{\cdot}6$ or $1{\cdot}6$; by $(1 - 1/n)\sin\alpha$ when $n = 1$. The value of n occurring in the tangent formula is the only one approximating to a real refractive index.

The possibility of a complex structure to take the place of the parallel plate is attractive but, I fear, not hopeful.

In reply to Mr Guild, I doubt whether there would be sufficient illumination. One wants to limit the range of wave-lengths to about 30 Ångström units and a narrow-band filter that will do this has generally a medium transmission of only about 10 to 15 per cent. Possibly such filters as Wratten Nos. 62, 74, or 75 might be suitable on a bright day. In the laboratory with a "Pointolite" source any one of these filters would be suitable.

A READING OF ELEMENTARY COLORIMETRIC THEORY

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MS. received, 2nd June, 1932.

ABSTRACT. The meanings to be assigned to the symbols =, +, and − in a colour equation, and the extent to which elementary algebraic manipulation is permissible, are considered. The nature of the colour triangle and of the variables used in colorimetry are discussed.

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1. It is well known that the judgments we make from the comparison of different visual sensations are very fallible. Our estimates of the relative brightness of different sources of the same type are usually inaccurate and inconstant. When the sources are not of the same type, so that differences of colour are present in addition to differences of brightness, the errors, already large, tend to be greatly magnified. Corresponding mistakes are made in the comparison of materials. We find that the sole reliable course is to secure a match for both brightness and colour. Only under these conditions will our eyes yield results that can be indefinitely repeated. For scientific purposes we introduce restrictions that are not usually thought necessary. From among the sets of conditions which have been found suitable for the consistent recognition of differences in the appearance of materials or sources, particular conditions (those for which the greatest amount of scientific information is available) have been selected and adopted in standardising laboratories and elsewhere. In the literature of colorimetry these standard conditions are to be assumed when colour matching is mentioned unless the contrary is specifically stated. These conditions are that the linear dimensions of the whole field in which the matching takes place subtend at the eye angles of about 2°, that this field is focused on the fovea centralis, and that the brightness of the surrounding field is small compared with that of the matching field itself. The two objects to be compared should subtend approximately equal solid angles at the eye, and the apparent separation between the two should be very fine, but visible. It is also necessary that the field brightness should lie in the range within which colour differences are easily seen.

2. We are accustomed to recognise two distinct aspects of colour. From an extensive collection of coloured materials we may select some sets showing changes of hue only, others exhibiting differences of shade but having the same hue, and still others exhibiting changes in both these characters. It might be thought that the series of sensations we experience on looking in turn at a set of the second kind, where the only difference of which we are conscious is in the shade of the colours, would be

of special significance in colorimetry. We must be on our guard against admitting such assumptions. Judgments of the kind indicated may be of psychological interest, but they lie outside the field of colorimetry. It is in fact known that an extended series of colours chosen in this way, and the simple colorimetric series with which it might well be expected to correspond, do not in general agree closely with one another. In colorimetry we restrict ourselves to complete matches.

3. Colorimetry would clearly be of little value if a match made by one person were not a match when viewed by many other persons. We have learnt that the great majority of mankind agree closely with one another in colour matching. There are exceptional groups whose colour vision differs markedly from that of the majority. Such persons are supposed to be excluded when colour measurements are carried out. Under the refined conditions secured in the best modern colorimeters, small differences are also found in the matches acceptable to individuals of the normal majority. We have therefore sometimes to distinguish between colour matches made by a particular individual and those which to the best of our knowledge represent the matches of an average normal individual. The existence of these minor individual deviations must not lead us to forget that the extent of the agreement is much more remarkable than that of the deviations. This fact makes it permissible to define, from the detailed examination of a somewhat small number of persons having the normal type of colour vision, a conventional average observer, for the purpose of establishing a metrical basis for colorimetry which shall be generally acceptable.

4. An experimental fact of importance enables a great simplification to be made in the classification and specification of colours. It is always possible to determine a precise colour match, in the restricted colorimetric sense which we shall henceforth imply when we speak of matches, such that the two colours will continue to be matched for the same observer whether his eyes be rested or fatigued: the match is also maintained as the radiation is varied in intensity over a large range, provided that in these variations the energy of the radiation is altered for all wave-lengths in the same proportion, and that, if two separate sources are used, this proportion is the same for both illuminants. This result enables us to exclude the absolute intensity of radiation as a factor in the measurements. We are thus not concerned with anything but relative intensities*.

5. *Colour equations.* Since in colorimetry we exclude everything but judgments that a match has or has not been attained, we are not in fact measuring or classifying colours themselves, but only the stimuli which evoke them. The experimental procedure invariably consists in projecting on two neighbouring portions of the retina, which we will conventionally designate L and R (for left and right) re-

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* Assertions contrary to this conclusion have sometimes been made. Where adequate experimental details have been given, these assertions have not been confirmed on repetition of the experiments at the National Physical Laboratory. Effects resembling some of those described can be obtained with an approximate, but not precise, colour match. Descriptions of some other experiments omit essential details, and leave us with the impression that their significance has not been appreciated. For example, in changing the intensity of illumination the number of methods it is legitimate to employ is very limited, but frequently nothing is said of the method actually adopted even when special apparatus for this purpose appears to have been necessary.

spectively, two stimuli, say **M** and **N**, so that **M** falls on *L* only and **N** on *R* only, and deciding whether the sensations they evoke can or cannot be distinguished from one another. For the most part we are only interested in the cases when **M** matches **N**, and we denote this occurrence by the colour equation

$$\mathbf{M} = \mathbf{N},$$

which is to be regarded as a conventional abbreviation of the statement "the stimulus **M** falling on the retinal region *L* gives rise to a sensation which matches that due to the falling of the stimulus **N** on the retinal region *R*." It is of course essential that the two sensations should be steady and should be experienced simultaneously*.

6, *Transposition*. By interchanging the sources, or by an equivalent procedure, we may cause the stimulus **M** to fall on *R* and **N** to fall on *L*. If under these conditions a match results we must, in accordance with our convention, write it in the form

$$\mathbf{N} = \mathbf{M}.$$

It is an experimental fact that for ordinary observers $\mathbf{M} = \mathbf{N}$ involves also $\mathbf{N} = \mathbf{M}$. It is at the least questionable† whether any observer for whom this law does not hold is fitted for colorimetric work. If we assume that we have only to deal with observers for whom the transposition law is true, we may, for conciseness, read $\mathbf{M} = \mathbf{N}$ as "**M** matches **N**," but we must not forget that this abbreviated statement has only a conventional meaning, for **M** and **N** are physical magnitudes and matching necessarily concerns sensations. It should be noted that the transposition law involves $\mathbf{N} = \mathbf{N}$, that is to say two physically indistinguishable stimuli falling on contiguous regions of the retina produce sensations which match one another.

7. *Addition and subtraction*. In nature any particular element of the retina receives light from only one object at a time. By optical means we may however cause light from more than one source to fall at the same time upon the same part of the retina‡. The two stimuli are then said to be added. Conversely, if from a compound or added stimulus a component is removed, the process is named subtraction. The general experimental law on the sensations evoked by added stimuli is

if $$\mathbf{M} = \mathbf{N} \text{ and } \mathbf{P} = \mathbf{Q}$$

then $$\mathbf{M} + \mathbf{P} = \mathbf{N} + \mathbf{Q};$$

the subtractive law

if $$\mathbf{P} = \mathbf{Q} \text{ and } \mathbf{M} + \mathbf{P} = \mathbf{N} + \mathbf{Q}$$

then $$\mathbf{M} = \mathbf{N}$$

is the converse of the additive law.

[*] This condition does not necessarily require the stimulus to be constant: for example, it may be regularly interrupted at very short intervals of time, or may consist of a series of stimuli of different types flashed at short intervals in regular succession on the retinal region concerned.

[†] I am indebted to Mr Guild for the observation that, in the standard routine, colorimetric measurements are strictly of the substitution type, so that asymmetry in the observer tends to be compensated. If the compensation were exact, an asymmetrical observer's results would not necessarily be inferior to those of a symmetrical observer provided a suitable routine, i.e. a routine in which experimental transposition is excluded, were followed.

[‡] We include regularly repeated successive vision of the various sources provided the period is short enough for a uniform sensation to be experienced.

If we take the component stimulus $\mathbf{P}$ from the compound stimulus $\mathbf{H}$ the resultant stimulus is obviously $\mathbf{H}-\mathbf{P}$. We may therefore write the subtractive law in the form

if $$\mathbf{P}=\mathbf{Q} \text{ and } \mathbf{H}=\mathbf{K}$$

then $$\mathbf{H}-\mathbf{P}=\mathbf{K}-\mathbf{Q}.$$

By repeating the processes of addition or subtraction several times we may extend these laws to stimuli compounded of many components. The addition law for example takes the form

if $$\mathbf{M}_h=\mathbf{N}_h \quad (h=1, 2, 3 \ldots k)$$

then $$\mathbf{M}_1+\mathbf{M}_2+\ldots+\mathbf{M}_k=\mathbf{N}_1+\mathbf{N}_2+\ldots+\mathbf{N}_k.$$

In the special case when all the $\mathbf{M}$ stimuli are physically alike and similarly all the $\mathbf{N}$ stimuli are alike we have

if $$\mathbf{M}=\mathbf{N}$$

then $$k\mathbf{M}=k\mathbf{N},$$

where k is an arbitrary integer.

By utilising the subtraction law and following a familiar process of reasoning we may extend this law to apply to any real finite non-negative value of k. This is the result already stated in Section 4. It is clear that the substantiation of the assertion mentioned in the footnote to that section would disprove the fundamental addition law of colorimetry.

8. *Equations with unreal stimuli.* From

$$\mathbf{H}-\mathbf{P}=\mathbf{K}-\mathbf{Q},$$
$$\mathbf{P}=\mathbf{P},$$
$$\mathbf{Q}=\mathbf{Q},$$

we derive by addition

$$\mathbf{H}=\mathbf{K}+\mathbf{P}-\mathbf{Q},$$
$$\mathbf{K}=\mathbf{H}-\mathbf{P}+\mathbf{Q},$$
$$\mathbf{H}+\mathbf{Q}=\mathbf{K}+\mathbf{P}.$$

These examples illustrate the applicability of simple algebraic laws to colour equations. It is however possible by the application of the algebraic laws to derive from

$$\mathbf{M}+\mathbf{P}=\mathbf{N}+\mathbf{Q}$$

such a result as

$$\mathbf{M}-\mathbf{Q}=\mathbf{N}-\mathbf{P}.$$

Now since $\mathbf{M}$ and $\mathbf{Q}$ may be chosen independently of one another, the stimulus $\mathbf{M}$ may not, in any physical sense, contain $\mathbf{Q}$ as a part; the stimulus denoted by $\mathbf{M}-\mathbf{Q}$ is then of a kind which can have no physical existence. Examples may arise in which the sensation supposed to be evoked by the stimulus $\mathbf{M}-\mathbf{Q}$ is also not a realisable sensation, and the question arises whether under these conditions the last equation is a legitimate statement. It is clearly inconvenient in applying formal processes to equations to have to consider whether every expression which is obtained at any stage of the work represents something which can actually be produced in a laboratory or be experienced by an observer, and the restriction is unnecessary if its absence can lead to no conclusions which contradict our experimental experience. If $\mathbf{Q}$ is not a component of $\mathbf{M}$ or $\mathbf{P}$ or $\mathbf{M}+\mathbf{P}$ and similarly $\mathbf{P}$ is not a component of $\mathbf{N}$ or $\mathbf{Q}$ or $\mathbf{N}+\mathbf{Q}$, the only meaning capable of experimental verification which can be assigned to $\mathbf{M}-\mathbf{Q}=\mathbf{N}-\mathbf{P}$ is $\mathbf{M}+\mathbf{P}=\mathbf{N}+\mathbf{Q}$. The unlimited application of the subtractive process to colour matches clearly cannot, in virtue of the laws to which we have already referred, lead to conclusions inconsistent with experimental results, and we shall therefore admit colour equations derived from experimental colour matches by the application of formal algebraic laws without restriction.

9. *The transitive law*. The practical applications of colorimetry largely rest on the experimental law *if* **M** *matches* **N** *and* **N** *matches* **P** *then* **M** *matches* **P**. For example if

$$\mathbf{M}=\mathbf{N},\ \mathbf{N}=\mathbf{P}+\mathbf{Q},\ \mathbf{Q}=\mathbf{R}+\mathbf{T},$$

we shall have

$$\mathbf{M}=\mathbf{P}+\mathbf{Q},$$

$$\mathbf{N}=\mathbf{P}+\mathbf{R}+\mathbf{T},$$

and

$$\mathbf{M}=\mathbf{P}+\mathbf{R}+\mathbf{T}.$$

It should be observed that for an asymmetrical observer we should either have to restrict the form in which equations can be written on application of the transitive law or else modify our original reading of a colour equation.

10. *Unit stimuli*. In Section 7 we expressed the result of adding together k stimuli each equal to $\mathbf{M}$ by $k\mathbf{M}$. In the development of colorimetry as a practical art it is necessary to make use of many stimuli of similar physical constitution but differing from one another in intensity or total energy. The relations of these stimuli to one another must be known, and it is therefore desirable to adopt some convenient reproducible stimulus as a unit and express all similar stimuli as multiples of this unit. Colorimeters are provided with means of measuring these multipliers. In general we shall use small italic letters to denote the multipliers, choosing the same letter of the alphabet as the black letter capital which denotes the unit stimulus. When no multiplier is recorded, unity is to be understood, in agreement with the usual algebraic convention.

11. *The character of normal colour vision*. We now come to the most remarkable of the experimental laws of colour matching. *Given any four stimuli, a colour match can always be obtained between suitable multiples of these stimuli*. Expressed in terms of a colour equation, if $\mathbf{W}$, $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{Z}$ are convenient units of the four stimuli, we can always find positive values of w, x, y, z to satisfy an equation of the form

$$w\mathbf{W}=x\mathbf{X}+y\mathbf{Y}+z\mathbf{Z},$$

or of the form

$$w\mathbf{W}+x\mathbf{X}=y\mathbf{Y}+z\mathbf{Z}.$$

It is convenient to regard the first of these two equations as the standard form. Obviously the colour equation for every match may be brought to this form so long as negative values of x, y, z are not excluded.

12. *Unit equations*. It is possible then to match any stimulus **W** with the sum of suitable multiples of three arbitrarily chosen stimuli **X**, **Y**, **Z**, provided that w is not zero. This proviso is clearly equivalent to the condition that it shall not be possible to effect a match between the three stimuli **X**, **Y**, **Z** alone. The fundamental method of colour measurement is derived directly from this law. The experimental equipment consists of means of producing steady radiation of three independent kinds, means for adding together any required multiples of selected units of these stimuli on either or both sides of a photometer field, and means of illuminating one side by the stimulus to be evaluated for colour. All the multiples of the three reference stimuli are adjusted until a match is obtained. The match (after algebraic transposition if necessary) is recorded by the standard colour equation

$$w\mathbf{W} = x\mathbf{X} + y\mathbf{Y} + z\mathbf{Z}.$$

As a rule no units will have been created for the stimulus **W**, and the only numerical factors of importance are the ratios of the three coefficients x, y, z. We may choose a unit for the stimulus **W** to effect any desired simplification in the equation. The convention adopted is to make $w = 1$ when $x + y + z = 1$; the resulting equation is called a unit colour equation, and the process of converting the equation to this form may be named unification.

13. As a result of applying algebraic processes to colour measurements we frequently arrive at colour equations of the form

$$x\mathbf{X} + y\mathbf{Y} + z\mathbf{Z} = 0,$$

where **X**, **Y** and **Z** are known to be colorimetrically independent. An equation of this type is always the equivalent of the three equations

$$x = 0, \quad y = 0, \quad z = 0.$$

The reduction of equations to the form stated is one of the simplest methods of solving colour equations.

14. *The trichromatic basis of colorimetry*. Physically light consists of radiations of many wave-lengths, and these several components are entirely independent of one another. We cannot for example relate the physical properties of light of one wave-length to those of light of three other wave-lengths. There is thus a marked distinction between the properties of light and the colour sensations light evokes. From a knowledge of the physical constitution of several stimuli we can say whether any of them, and if so which, will give rise to sensations which match one another. On the other hand, from the fact that two stimuli give rise to like colour sensations we can infer no physical correspondence. The fact that many physically dissimilar stimuli give rise to like sensations shows that if we all wish to express the results of colour comparisons in the same terms—necessarily in terms of stimuli—we must arbitrarily choose to specify one out of the multiplicity of stimuli which give rise to the particular sensation. Much the most important way of selecting a reference system is to choose three stimuli that can be accurately reproduced in any laboratory, to devise a method of fixing the three units, and to construct the unit equation for any stimulus in terms of these three stimuli. This equation usually has to be derived

indirectly, because it is usually either inconvenient or impracticable to employ the reference stimuli in the normal experimental routine of matching. The three reproducible stimuli are therefore replaced in routine work by three stimuli peculiar to the individual instrument, and the connection between these three instrumental stimuli and the absolute standard stimuli is determined at a standardising laboratory.

15. We have mentioned in Section 12 the essential features of a satisfactory colorimeter. The description of routine measurements which follows applies to the Guild colorimeter, which meets these conditions in a simple way. The changes necessary when other instruments are used should be obvious.

Perhaps the kind of measurement most frequently required is the specification of the colour of a card coated with a certain pigment when illuminated by a given standard source. The first step is to match the light reflected from the card with light (from a steady source fixed in the instrument) which has traversed three stable coloured filters. The amount of light which passes each of the three filters can be controlled by three opaque shutters, and the amounts exposed are read directly on scales attached to the shutters. The colours of the three filters are red, green, and blue, and the stimuli corresponding to unit opening of each of the three shutters in turn will be denoted by $\mathbf{I}'$, $\mathbf{J}'$, $\mathbf{K}'$. Considering only the simplest case, it will be found that, when one half of the photometer field is illuminated by light from the specimen, a match is secured when the other half receives light from each of the three filters, the shutter readings being i, j, k respectively. The colour equation then takes the form

$$\mathbf{C}_1 = i_1\mathbf{I}' + j_1\mathbf{J}' + k_1\mathbf{K}',$$

where $\mathbf{C}$ denotes the colour reflected from the card and the numerical suffix indicates that we have not yet settled any units for the colour under measurement. The lamp fitted into the colorimeter is assumed to give a steady output, but is not necessarily a carefully calibrated standard lamp run under precisely determined conditions. Variations in its brightness will cause small relative changes in the temporary units we have denoted by $\mathbf{I}'$, $\mathbf{J}'$, $\mathbf{K}'$. If the filters are carefully chosen for the purpose, these changes can be confined to brightness variations, the colour being constant. To eliminate these variations, a reading is obtained of the colour of the light received directly from the standard lamp. Denoting this by $\mathbf{S}$, we obtain the colour match

$$\mathbf{S}_0 = i_0\mathbf{I}' + j_0\mathbf{J}' + k_0\mathbf{K}'.$$

This completes the routine observations required in a simple test.

16. On account of the variations we have mentioned, the units $\mathbf{I}', \mathbf{J}', \mathbf{K}'$, though constant over short periods of time—such as a day or half a day—must be regarded as variables. Not only do they vary in absolute magnitude but they alter relatively to one another. There is therefore no stability in the experimental scale readings i_0, j_0, k_0 and i_1, j_1, k_1. In terms of stable units we have equations of the form

$$\mathbf{S} = i_s\mathbf{I} + j_s\mathbf{J} + k_s\mathbf{K}$$

and

$$\mathbf{C} = i_c\mathbf{I} + j_c\mathbf{J} + k_c\mathbf{K}.$$

Now $\mathbf{S}$ and $\mathbf{S}_0$ denote different amounts of physically similar radiation, so that a colour match $\mathbf{S}=\sigma\mathbf{S}_0$ certainly holds. By the transitive law we then obtain

$$(i_s\mathbf{I}-\sigma i_0\mathbf{I}')+(j_s\mathbf{J}-\sigma j_0\mathbf{J}')+(k_s\mathbf{K}-\sigma k_0\mathbf{K}')=0.$$

Now the quantities in each of the brackets denote amounts of radiation physically similar to $\mathbf{I}$, $\mathbf{J}$, and $\mathbf{K}$ respectively, three colorimetrically independent stimuli. By the principle of Section 13 we therefore have

$$i_s\mathbf{I}=\sigma i_0\mathbf{I}',\quad j_s\mathbf{J}=\sigma j_0\mathbf{J}',\quad k_s\mathbf{K}=\sigma k_0\mathbf{K}'.$$

In exactly the same way, since $\mathbf{C}$ and $\mathbf{C}_1$ by hypothesis denote different amounts of physically similar radiation, there exists a colour match $\mathbf{C}=\gamma\mathbf{C}_1$, leading to the three equations

$$i_c\mathbf{I}=\gamma i_1\mathbf{I}',\quad j_c\mathbf{J}=\gamma j_1\mathbf{J}',\quad k_c\mathbf{K}=\gamma k_1\mathbf{K}'.$$

The two sets of equations give two expressions for each of the ratios $\mathbf{I}/\mathbf{I}'$, $\mathbf{J}/\mathbf{J}'$, $\mathbf{K}/\mathbf{K}'$. Equating the two expressions, we have

$$i_c=\frac{\gamma}{\sigma}\cdot\frac{i_s}{i_0}i_1,\quad j_c=\frac{\gamma}{\sigma}\cdot\frac{j_s}{j_0}j_1,\quad k_c=\frac{\gamma}{\sigma}\cdot\frac{k_s}{k_0}\cdot k_1.$$

Thus the stable coefficients i_c, j_c, k_c can be found apart from the common unknown factor γ/σ. As a rule we desire to obtain the unit equation for $\mathbf{C}$ in terms of $\mathbf{I}$, $\mathbf{J}$, $\mathbf{K}$. The condition

$$i_c+j_c+k_c=1$$

is thus to be satisfied. If then we write

$$i'_c=\frac{i_s}{i_0}i_1,\quad j'_c=\frac{j_s}{j_0}j_1,\quad k'_c=\frac{k_s}{k_0}k_1,$$

we find

$$i_c=i'_c/e,\quad j_c=j'_c/e,\quad k_c=k'_c/e,$$

where

$$e=i'_c+j'_c+k'_c.$$

17. In normal routine the procedure is rendered as easy as possible by choosing

$$\mathbf{S}=\frac{1}{3}\mathbf{I}+\frac{1}{3}\mathbf{J}+\frac{1}{3}\mathbf{K}$$

as the unit equation of the standard illuminant on the instrumental system. We may then write

$$i'_c=\frac{i_1}{i_0},\quad j'_c=\frac{j_1}{j_0},\quad k'_c=\frac{k_1}{k_0},$$

or alternatively

$$i'_c=i_1,\quad j'_c=\frac{i_0}{j_0}j_1,\quad k'_c=\frac{i_0}{k_0}k_1,$$

and determine the unit equation of the colour $\mathbf{C}$ as before from

$$i_c=i'_c/e,\quad j_c=j'_c/e,\quad k_c=k'_c/e,$$

where

$$e=i'_c+j'_c+k'_c.$$

18. The simple procedure outlined in Section 15 fails when the three coefficients i_1, j_1, k_1 are not all positive. With a suitable choice of filters this rarely occurs in technical colorimetry, but the change of procedure required is simple. When we cannot obtain a direct match with the test colour alone, we admit an additional

stimulus derived from the colorimeter lamp to the side of the photometer field where the test specimen is seen. Call this additional stimulus **A**. We then secure the match

$$\mathbf{C}_1 + \mathbf{A} = i_2\mathbf{I}' + j_2\mathbf{J}' + k_2\mathbf{K}'.$$

By merely obscuring the specimen we obtain the additional match

$$\mathbf{A} = i_3\mathbf{I}' + j_3\mathbf{J}' + k_3\mathbf{K}'.$$

Applying the subtraction law we find

$$\mathbf{C}_1 = i_1\mathbf{I}' + j_1\mathbf{J}' + k_1\mathbf{K}',$$

where

$$i_1 = i_2 - i_3,$$
$$j_1 = j_2 - j_3,$$
$$k_1 = k_2 - k_3.$$

If the additional stimulus **A** is the natural light from the lamp of the instrument, i_3, j_3, k_3 will all be positive. Frequently the additional light is caused to traverse a filter similar to those used in the main portion of the instrument. If the filters correspond exactly, two of the three quantities i_3, j_3, k_3 will be zero.

19. The determination of a colour **C** by means of the coefficients i_c, j_c, k_c is sufficient for many purposes when all measurements are carried out on the one instrument to which **I**, **J**, **K** relate. To make them comparable with measurements carried out on other instruments, particularly instruments in other laboratories or workshops, the instrumental stimuli must be replaced by agreed standard stimuli. At present the only standards of international standing are those recommended by the Commission Internationale de l'Éclairage at Cambridge in 1931*. The reference stimuli on this system are denoted by **X**, **Y**, **Z**, and the calibration of colorimeters at the National Physical Laboratory is given in the form

$$\mathbf{I} = x_i\mathbf{X} + y_i\mathbf{Y} + z_i\mathbf{Z},$$
$$\mathbf{J} = x_j\mathbf{X} + y_j\mathbf{Y} + z_j\mathbf{Z},$$
$$\mathbf{K} = x_k\mathbf{X} + y_k\mathbf{Y} + z_k\mathbf{Z},$$

where **I**, **J**, **K** represent units of the instrumental stimuli in such proportions that equal amounts match the standard stimulus **S**, as illustrated in Section 17. To express the colour on the international system we have only to substitute in the instrumental equation

$$\mathbf{C} = i_c\mathbf{I} + j_c\mathbf{J} + k_c\mathbf{K}$$

the values given in the calibration equations for **I**, **J**, **K** to obtain a colour equation between **C** and the international units **X**, **Y**, **Z**. When the numerical work has been performed we are left with an equation of the form

$$\mathbf{C}' = x'\mathbf{X} + y'\mathbf{Y} + z'\mathbf{Z},$$

where the accents denote that the equation is not yet unified. The final result is

$$\mathbf{C} = x\mathbf{X} + y\mathbf{Y} + z\mathbf{Z},$$

where

$$\frac{x}{x'} = \frac{y}{y'} = \frac{z}{z'} = \frac{1}{x' + y' + z'}.$$

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* T. Smith and J. Guild, *Trans. Opt. Soc.* **33** 1931–32 (73).

20. The perusal of numerous papers on colorimetric theory might lead a reader to the conclusion that the colour which yields a match represented by

$$\mathbf{C} = \frac{1}{3}\mathbf{R} + \frac{1}{3}\mathbf{G} + \frac{1}{3}\mathbf{B},$$

where **R**, **G**, and **B** are the units of a colorimetric system, is of very unusual significance. This conclusion would be incorrect. It happens that the sources which have been employed for the determination of the units have most frequently been assigned these coefficients, and that, as a source yielding a continuous spectrum has been preferred for this purpose, various experimenters have adopted a source which appeared to them a reasonable representation of a "white" source. The particular colour represented by this point has thus often been called white to the exclusion of other colours, and it might easily be inferred that special properties belong to a system in which these coefficients are assigned to white. The fact is that white, like any other common designation such as yellow or green, is applied to a somewhat extensive group of colours. There is no evidence that, given a range of colours of this group from which to select, a given observer will always or indeed often select the same colour as the nearest approach to his idea of subjective white. On the contrary, there is evidence of systematic differences in psychological questions of this kind between individuals whose colour vision is normal. A given source will be classed by one observer as somewhat yellower than white, and by another as somewhat bluer. The conclusion is that our personal views on these matters are not of the least significance in colorimetry. It is a mere accident that satisfactory systems have resulted from assigning equal co-ordinates to any of these stimuli. The criteria for a satisfactory system for the numerical specification of colours are of a quite different kind. They have been carefully considered in constructing the international trichromatic system. Whether the system has this or that special feature is not important. What is important is that all colorimetrists should use the same system.

21. *The colour triangle.* The literature of colorimetry also abounds with references to the colour triangle. This is merely a triple co-ordinate system for the graphical representation of colour measurements. Since the unit equation of any colour **C** is of the form

$$\mathbf{C} = x\mathbf{X} + y\mathbf{Y} + z\mathbf{Z},$$

where $x + y + z = 1$, any magnitude **C** given by an equation of this type can be conveniently represented by the areal co-ordinates x, y, z of a point. Let XYZ be any plane triangle, and C be any point in this plane. Let the areas of the triangles XYZ, CYZ, CZX, CXY be denoted by Δ, ξ, η, ζ respectively, where Δ is always considered positive, ξ positive if C and X are on the same side of YZ, η positive if C and Y lie on the same side of ZX, and ζ positive if C and Z are situated on the same side of XY. Then obviously

$$\xi + \eta + \zeta = \Delta,$$

and the point C may be used to represent the colorimetric properties of the stimulus **C** if X, Y, Z represent those of the stimuli **X**, **Y**, **Z** respectively and

$$\frac{\xi}{x} = \frac{\eta}{y} = \frac{\zeta}{z} = \Delta.$$

It should be particularly noticed that the shape of the triangle is immaterial. In the past an equilateral triangle was most frequently employed, but it is more generally convenient to employ a right-angled triangle, for ordinary graph paper may then be used in plotting points on the chart. In this use of ruled paper we depend upon the theorem that the areas of triangles on the same base are proportional to their heights. Thus, in the use of ordinary graph paper, points are plotted exactly as with ordinary cartesian co-ordinates, the variables used corresponding to the letters at the acute angles of the triangle. The centroid of the triangle, whose distance from any side is one-third of the distance of the corresponding vertex, is clearly the point having the co-ordinates $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$.

It is somewhat curious that the nature of the co-ordinates used in the colour triangle is invariably mis-described. They appear to have always been referred to as trilinear co-ordinates. The distinction is merely one of name when an equilateral triangle is adopted as the basis of a chart, but with other triangles trilinear co-ordinates have not the properties desired. Trilinear co-ordinates represent the perpendicular distances of a point from the three sides of a triangle. In trilinears the point with the co-ordinates $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ is thus the incentre, not the centroid. Charts plotted in trilinear co-ordinates properly so called are not of colorimetric importance.

22. We have observed that a colour match can be made between any four colours; in general, four colours are necessary, but in some cases three are sufficient through the vanishing of the coefficient of one stimulus.

It is of importance to know what stimuli can be matched by a mixture of two given stimuli, and the colour triangle owes much of its value to the fact that the solution to this problem assumes a particularly simple form. The chart also shows how many colorimetric units of the two given stimuli are needed to match one colorimetric unit of the third. Let $\mathbf{C}_1$ and $\mathbf{C}_2$ be trichromatic units of two given stimuli on the system in which $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{Z}$ are the unitary stimuli, and suppose that c_1 units of $\mathbf{C}_1$ and c_2 units of $\mathbf{C}_2$ together match one unit of $\mathbf{C}_3$. Then

$$\mathbf{C}_3 = c_1\mathbf{C}_1 + c_2\mathbf{C}_2.$$

But

$$\mathbf{C}_p = x_p\mathbf{X} + y_p\mathbf{Y} + z_p\mathbf{Z} \quad (p = 1, 2, 3),$$

so that

$$x_3\mathbf{X} + y_3\mathbf{Y} + z_3\mathbf{Z} = (c_1x_1 + c_2x_2)\,\mathbf{X} + (c_1y_1 + c_2y_2)\,\mathbf{Y} + (c_1z_1 + c_2z_2)\,\mathbf{Z},$$

and since $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{Z}$ are independent stimuli, we must have

$$x_3 = c_1x_1 + c_2x_2,$$
$$y_3 = c_1y_1 + c_2y_2,$$
$$z_3 = c_1z_1 + c_2z_2.$$

Now the equation of any straight line in areal co-ordinates is a homogeneous equation

$$lx + my + nz = 0.$$

The preceding equations show that if this straight line passes through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) it also passes through (x_3, y_3, z_3). In other words, all stimuli which can be matched by the stimuli represented by the points $\mathbf{C}_1$ and $\mathbf{C}_2$ are represented on the chart by the straight line through $\mathbf{C}_1$ and $\mathbf{C}_2$.

Since the equations are in unit form, for each point $x + y + z = 1$. Adding the three equations together we find $c_1 + c_2 = 1$. Introducing this on the left side, we have

$$\frac{c_1}{x_2 - x_3} = \frac{c_2}{x_3 - x_1} = \frac{1}{x_2 - x_1},$$

with similar equations for the y and z co-ordinates. Thus, in terms of distances measured on a colour triangle,

$$\frac{c_1}{C_2C_3} = \frac{c_2}{C_3C_1} = \frac{1}{C_2C_1}.$$

It should be realised that the colour chart and the unit equations represent precisely the same facts. As a rule the equations are of greater convenience where much calculation is needed, and the chart is most useful in presenting a pictorial representation of the colorimetric relations between various stimuli, which can thus be appreciated more readily than by tabulating large numbers of figures.

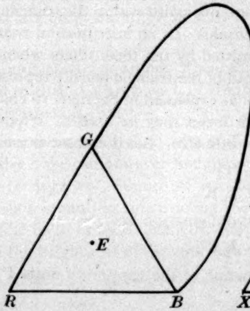

Fig. 1.

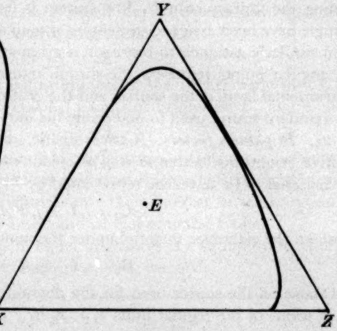

Fig. 2.

23. The expression we have found connecting C_1, C_2, C_3 shows that C_3 is the centre of gravity of particles of masses c_1 and c_2 placed at C_1 and C_2 respectively. This parallel can easily be extended to the case when any number of components on the one hand are matched with a given stimulus on the other. Now any real stimulus is composed of radiation of different wave-lengths. From an analysis of its physical constitution and a knowledge of the colorimetric units of all spectral radiation we can express a colorimetric unit of the given stimulus in terms of the colorimetric units of spectral colours. Every spectral colour is represented either by a zero or by a positive coefficient c. Thus the point representing any real colour must lie within the area of the colour triangle marked out by the spectral locus. Fig. 1 shows the spectral locus when the unitary stimuli are light of wave-lengths 700, 546·1, and 435·8$m\mu$ respectively. The first represents the red end of the visible spectrum, and the other two are the strong green and blue lines of the mercury spectrum. This

triangle bears a close resemblance to those of some instrumental systems. All spectral colours lie outside the triangle, and so have at least one negative colour co-ordinate. On the other hand most industrial colours are represented by points within the triangle, and so have all their co-ordinates positive.

Fig. 2 represents the international standard colour triangle. The spectral locus lies within the triangle, so that every possible real colour has all its co-ordinates positive on this system. It will be inferred from what has been said already that all real colours are represented by points in the area bounded by the spectral curve and by the straight line joining its extremities. In the international system none of the unitary stimuli are real.

Although the colour chart is usually referred to as the colour triangle, this description is misleading. The triangle merely gives the units and the points representing the unitary stimuli. The system is fixed when the angular points of the triangle have been assigned to definite stimuli and one point within the triangle has been similarly assigned to represent a given stimulus. In an instrumental triangle the angular points represent the stimuli transmitted by the three filters when the instrumental lamp is the source, and the centroid of the triangle usually represents the standard source used to determine the units as explained in Sections 16 and 17.

24. *Brightness factors*. A trichromatic colorimeter may be used to determine relative brightness factors as well as colour co-ordinates. Let the observations on the material to be tested be represented by

$$\mathbf{C}_1 = i_1\mathbf{I}' + j_1\mathbf{J}' + k_1\mathbf{K}',$$

those on the reference material under the same illumination by

$$\mathbf{R}_2 = i_2\mathbf{I}' + j_2\mathbf{J}' + k_2\mathbf{K}',$$

and those on the source used for the determination of the temporary units $\mathbf{I}'$, $\mathbf{J}'$, $\mathbf{K}'$ in terms of the regular units $\mathbf{I}$, $\mathbf{J}$, $\mathbf{K}$ by

$$\mathbf{S}_0 = i_0\mathbf{I}' + j_0\mathbf{J}' + k_0\mathbf{K}'.$$

The regular units $\mathbf{I}$, $\mathbf{J}$, $\mathbf{K}$ have luminosities assigned to them in the calibration of the instrument. Let them be β_i, β_j, β_k. Then, if

$$\mathbf{S} = i_s\mathbf{I} + j_s\mathbf{J} + k_s\mathbf{K}$$

is the standard equation for the source, we have seen that

$$\mathbf{I}' = \frac{ci_s}{i_0}\mathbf{I}, \quad \mathbf{J}' = \frac{cj_s}{j_0}\mathbf{J}, \quad \mathbf{K}' = \frac{ck_s}{k_0}\mathbf{K},$$

where c is some constant. Substituting these values in the first two equations we obtain

$$\mathbf{C}_1 = c\left\{\frac{i_s}{i_0}i_1\mathbf{I} + \frac{j_s}{j_0}j_1\mathbf{J} + \frac{k_s}{k_0}k_1\mathbf{K}\right\},$$

$$\mathbf{R}_2 = c\left\{\frac{i_s}{i_0}i_2\mathbf{I} + \frac{j_s}{j_0}j_2\mathbf{J} + \frac{k_s}{k_0}k_2\mathbf{K}\right\}.$$

Now a match involves equality of brightness as well as of colour. The two equations we have written down state that the brightnesses of the stimuli $\mathbf{C}_1$ and $\mathbf{R}_2$ measured on the instrument are equal to those of certain units each of brightness β_i, together with a certain number each of brightness β_j, and another number each of brightness β_k. The brightness of $\mathbf{C}_1$ relative to $\mathbf{R}_2$ is therefore given by

$$\frac{i_1 \frac{i_s}{i_0} \beta_i + j_1 \frac{j_s}{j_0} \beta_j + k_1 \frac{k_s}{k_0} \beta_k}{i_2 \frac{i_s}{i_0} \beta_i + j_2 \frac{j_s}{j_0} \beta_j + k_2 \frac{k_s}{k_0} \beta_k}.$$

It is important to observe that the brightness factor we are concerned with here has nothing to do with the luminosity of the colorimetric unit of a stimulus which matches **C**.

25. The luminosity of the colorimetric units naturally depends upon the trichromatic system concerned. The international system with units denoted by **X**, **Y**, **Z** is that of greatest importance. Let β_x, β_y, β_z be numbers proportional to the luminosity factors of these units—the scale on which they are chosen is unimportant, for we are only concerned with their relative magnitudes. Let

$$\mathbf{C} = x\mathbf{X} + y\mathbf{Y} + z\mathbf{Z}$$

be a unit colour equation. Then β_c, the brightness of the trichromatic unit **C**, is equal to that of x units each of brightness β_x, y units of brightness β_y, and z units of brightness β_z, i.e.

$$\beta_c = x\beta_x + y\beta_y + z\beta_z.$$

So long as we are dealing with real stimuli the brightness of the unit is necessarily positive. We can however find hypothetical unit stimuli for which the brightness is zero or negative. Stimuli of zero brightness owe their interest to the selection of two such stimuli in constructing the International Colorimetric Units.

The locus of all colorimetric units of equal brightness is a straight line. For, if β be the brightness of the unit represented by x, y, z, we have

$$\beta = x\beta_x + y\beta_y + z\beta_z,$$

or, since

$$x + y + z = 1,$$

$$x(\beta_x - \beta) + y(\beta_y - \beta) + z(\beta_z - \beta) = 0,$$

which is the standard form for the equation of a straight line in areal co-ordinates. It is easy to show that as β is varied this equation represents a family of parallel straight lines. Thus in the colour triangle the loci of unit stimuli of equal brightness are parallel straight lines. This result is also obvious on physical grounds.

The representative locus of stimuli of zero brightness is named the alychne. In the international system the side *XZ* of the triangle coincides with the alychne so that $\beta_x = \beta_z = 0$. We may arbitrarily put $\beta_y = 1$; the brightness of the trichromatic unit on this basis is then represented by the co-ordinate y.

26. In the foregoing discussion we have referred to variables such as i, j, k; r, g, b; or x, y, z as co-ordinates. This name is justified by their interpretation on the colour triangle. If we prefer physico-psychical terms we may call them colour valencies. Thus, if

$$\mathbf{C} = r\mathbf{R} + g\mathbf{G} + b\mathbf{B},$$

r, g, and b are respectively the red-, green- and blue-valencies of the stimulus **C**. When unreal units **X**, **Y**, **Z** are used, we may refer to x, y, z as the **X**-, **Y**-, **Z**-valencies.

THE EFFECT OF VARIATION IN THE FILTERS ON THE CALIBRATION COEFFICIENTS OF COLORIMETERS

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MS. received, 6th June, 1932.

ABSTRACT. Variations in the transmissions of the filters for light of any given wavelength alter the calibration constants of a colorimeter, and expressions are developed for these changes in order to ascertain what accuracy is necessary in spectrophotometric tests to yield ample accuracy in the calibration. The formula obtained is suitable for use for a considerable length of the spectrum, and involves only instrumental quantities utilised in the computation of the normal calibration coefficients.

When colorimeters are calibrated by the indirect method it is desirable to know the importance of variations in the transmission coefficients of the filters for light of different wave-lengths. This information enables useless additions to the cost of testing an instrument, arising from the determination in particular parts of the spectrum of transmission factors to an accuracy of no ultimate significance, to be avoided. The most important parts of the transmission curves from this point of view are the long wave-length ends of the curves for the green and blue filters. As is well known a very small amount of light, as it is appreciated visually, is transmitted by many filters. It is clear that the green and blue filters of a colorimeter may transmit light from the red end of the spectrum which is measurable without being sufficiently great to affect the co-ordinates of colours measured by the instrument. In this case work spent on the accurate determination of the transmissions of these filters at the red end of the spectrum is useless. On the other hand, it is easy to see that with a somewhat higher transmission this red light may be of real importance, and measurement to a certain degree of accuracy is essential. In the absence of stringent time limits the spectrophotometry can be carried out to an accuracy which far surpasses the needs of the colorimetrist, so that the problem is of real importance.

Let E denote the energy distribution of the source used to illuminate the filters of the instrument, V the standard visibility function, τ the transmission of a filter, and x a trichromatic co-ordinate, on the international system, of spectral light. The calibration of the instrument requires the determination of the integrals $\int \tau E V x d\lambda$ throughout the spectrum. In practice, the integrals are replaced by the sums of a finite number of elements

$$m_{pq} = \Sigma \tau_p E V x_q$$

taken at equal intervals of λ over the range for which V does not vanish. By this notation we distinguish a particular filter by the suffix p, and a particular colorimetric co-ordinate by the suffix q. The instrumental units corresponding to the three filters will be denoted by the letter F, and those of the international system by X. In the expressions which follow F and X always denote matrices of a single column. Square matrices will be denoted by Roman letters, diagonal matrices by Greek capitals, and linear matrices by small Greek letters. If the linear matrices have a single row the plain letter is used: if a single column an accent is added.

With this notation and the definition already given of the m's we have

$$F = \Delta m X,$$

where Δ is introduced to enable us to relate the units F and X correctly. If these units are to be so chosen that the stimulus with co-ordinates ϕ_0 on the instrumental system has co-ordinates ξ_0 on the standard system,

$$\phi_0 F = \xi_0 X,$$

or

$$\xi_0 = \phi_0 \Delta m,$$

or

$$\delta = \xi_0 M,$$

where M is the inverse of m and δ is the linear matrix whose elements are the products of corresponding elements of ϕ_0 and Δ. This equation therefore determines Δ, and the calibration of the instrument is thus obtained in the form

$$F = nX,$$

where $n = \Delta m$.

Now suppose that the transmissions of all the filters are changed for a wavelength λ corresponding to the co-ordinates ξ_λ on the standard system. In place of m we then have

$$\overline{m} = m - \epsilon'\xi_\lambda,$$

where ϵ' represents the diminution in τEV for the three filters. The inverse of $m - \epsilon'\xi_\lambda$ is

$$\begin{aligned}\frac{1}{m - \epsilon'\xi_\lambda} = \frac{M}{1 - \epsilon'\xi_\lambda M} &= M + M\epsilon'\xi_\lambda M + M\epsilon'\xi_\lambda M\epsilon'\xi_\lambda M + \dots \\ &= M + M\epsilon'\,[1 + \xi_\lambda M\epsilon' + (\xi_\lambda M\epsilon')^2 + \dots]\,\xi_\lambda M \\ &= M + \frac{M\epsilon'\xi_\lambda M}{1 - \xi_\lambda M\epsilon'},\end{aligned}$$

since $\xi_\lambda M\epsilon'$ is scalar. The new value of δ is thus given by the equation

$$\overline{\delta} = \xi_0 M + \frac{\xi_0 M\epsilon'}{1 - \xi_\lambda M\epsilon'}\,\xi_\lambda M,$$

where the fraction is a scalar factor. The new calibration coefficients are therefore given by

$$\overline{n}_{pq} = \left(\xi_0 \mu'_p + \frac{\xi_0 M\epsilon'}{1 - \xi_\lambda M\epsilon'}\,\xi_\lambda \mu'_p\right)\overline{m}_{pq},$$

where μ'_p denotes the pth column of M.

Now let a colour be found by measurement on the instrument to have co-ordinates ϕ on the instrumental system. If ι' denotes a single column matrix all of whose elements are unity the corresponding co-ordinates on the standard system are ξ, where

$$\bar{\xi} = \frac{\phi\bar{n}}{\phi\bar{n}\iota'} = \frac{\phi n + \phi\,(\bar{n} - n)}{\phi n\iota' + \phi\,(\bar{n} - n)\,\iota'},$$

or very approximately

$$\bar{\xi} = \xi + \frac{\phi\,(\bar{n} - n)}{\phi n\iota'} - \frac{\phi\,(\bar{n} - n)\,\iota'}{\phi n\iota'}\,\xi.$$

The last term merely serves to keep the sum of the coefficients unity.

From this expression it is clear that the largest changes in co-ordinates will occur when the largest terms are found in $\phi\,(\bar{n} - n)$. Now, by the procedure we have followed we have made $\phi_0\,(\bar{n} - n) = 0$, so that the change in the filters produces no effect in the co-ordinates of the source, and the differences in the neighbourhood of the source will necessarily be small. The character of the effects elsewhere can best be appreciated if particular cases are chosen. Suppose the transmission of the blue filter altered at the red end of the spectrum. From the values of the co-ordinates on the international system we see that the blue coefficients* in the calibration equations will not be affected, while the red coefficients will be affected about three times as much as the green. The coefficients of the green filter hardly alter: the red coefficient of the blue filter increases, and if (as will usually be the case) the co-ordinates ϕ_0 have all the same sign, the red coefficient of the red filter falls. The greatest changes will thus be found for colours which have their red and blue valencies on the instrumental system of opposite sign—if ϕ_0 represents the centroid, the difference in these elements of ϕ will serve as an approximate measure of the resulting change. In a similar way the effect of alterations in the red transmission of the green filter will be greatest for colours which have a large difference between the red and green valencies on the instrumental system.

From the formulae which have been developed here it is easy to evaluate the changes in co-ordinates in the part of the real colour field most sensitive to these alterations. Comparison with the variations in setting for matching a known colour in this part of the field enables the importance of the alteration to be determined.

At the red end of the spectrum the trichromatic co-ordinates change slowly, and it becomes possible to consider the whole of the light transmitted in this neighbourhood as if it were of the extreme red wave-length. Also, as the spectral locus on the colour triangle is nearly straight from red to yellow, light transmitted over the whole of this region may be represented with ample accuracy by light of a single suitably chosen wave-length. The formula as it has been given is thus sufficient for the most important problem for which it is required. Any other cases could readily be dealt with by superposition.

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* This expression is used to avoid clumsy circumlocution; the meaning attached to it should be obvious.

NOTE ON THE CORRECTION OF THE LUMINOSITIES OF COLORIMETRIC UNITS

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MS. received, 2nd June, 1932.

ABSTRACT. The trial-and-error method of correcting observed luminosities so as to be consistent with the standard visibility function may be replaced by the direct method described.

When a colorimetric system is constructed by the direct method, the evaluation on an experimental basis of the relative luminosity factors of the three stimulus units is not free from difficulty. The results obtained are those for individual observers, and their visibility functions will deviate to some extent from the accepted standard visibility function. It has been the custom to check the results obtained from experiment by calculations of the type illustrated below. When calculation and observation fail to agree it appears from published descriptions to have been thought that a trial-and-error method was the most suitable way of exploring the effect of different modifications in these factors; calculations are supposed to be carried out on this basis until agreement is reached. The purpose of this note is to show that this process of trial and error can with advantage be replaced by a direct determination of the necessary corrections to the luminosity factors.

The principle of the calculations is as follows: let V be the standard visibility function, and E the known energy distribution function of a real source of light. Then the colour equation of light of wave-length λ being

$$C = xX + yY + zZ,$$

the brightness of this colorimetric unit is

$$x_\lambda \beta_x + y_\lambda \beta_y + z_\lambda \beta_z,$$

where β_x, β_y, β_z are the luminosities of the units X, Y, Z respectively. The number of colorimetric units in the light given out by the source between the wave-lengths λ and $\lambda + d\lambda$ is represented by

$$\frac{E_\lambda V_\lambda d\lambda}{x\beta_x + y\beta_y + z\beta_z} \equiv \frac{E_\lambda V_\lambda d\lambda}{\beta_\lambda},$$

and the corresponding numbers of X, Y, and Z units are

$$\frac{x_\lambda E_\lambda V_\lambda d\lambda}{\beta_\lambda}, \quad \frac{y_\lambda E_\lambda V_\lambda d\lambda}{\beta_\lambda}, \quad \frac{z_\lambda E_\lambda V_\lambda d\lambda}{\beta_\lambda}.$$

Integrating we find that the colorimetric co-ordinates of the source are proportional to

$$\int \frac{x_\lambda E_\lambda V_\lambda d\lambda}{\beta_\lambda}, \quad \int \frac{y_\lambda E_\lambda V_\lambda d\lambda}{\beta_\lambda}, \quad \int \frac{z_\lambda E_\lambda V_\lambda d\lambda}{\beta_\lambda},$$

where integration extends over the wave-length range for which V does not vanish. If these three integrals are in the same ratio as the corresponding experimentally observed colour co-ordinates, the accuracy of the relative values of the β's is confirmed. If the ratios do not agree, corrected values for the β's must be found.

Since we are only concerned with the relative values of β_x, β_y, β_z we may assign an arbitrary value to one of them. If the unitary stimuli are real, the brightness of no unit can be a vanishingly small quantity. We may then assume $\beta_x = 1$: β_y and β_z, which represent the approximate relative luminosities of the other units found by experiment, are then definite numbers. We suppose that the correct values are in the ratio $1 : \beta_y + \delta\beta_y ; \beta_z + \delta\beta_z$. Put

$$\frac{x}{\xi} = \frac{y}{\eta} = \frac{z}{\zeta} = x + y\beta_y + z\beta_z,$$

$$t_{p,q} = \int \eta^p \xi^q E_\lambda V_\lambda d\lambda,$$

and let x_s, y_s, z_s be the trichromatic co-ordinates of the source whose energy function is E. Then $\delta\beta_y$ and $\delta\beta_z$ are to be found from the equations

$$cx_s = \int \frac{xEV d\lambda}{x + y(\beta_y + \delta\beta_y) + z(\beta_z + \delta\beta_z)},$$

$$cy_s = \int \frac{yEV d\lambda}{x + y(\beta_y + \delta\beta_y) + z(\beta_z + \delta\beta_z)},$$

$$cz_s = \int \frac{zEV d\lambda}{x + y(\beta_y + \delta\beta_y) + z(\beta_z + \delta\beta_z)},$$

where c is a constant of no importance. The three equations clearly give

$$c\{x_s + y_s(\beta_y + \delta\beta_y) + z_s(\beta_z + \delta\beta_z)\} = t_{0,0}.$$

Now if we expand by Taylor's theorem we obtain

$$cy_s = t_{1,0} - \delta\beta_y t_{2,0} - \delta\beta_z t_{1,1} + (\delta\beta_y)^2 t_{3,0} + 2\delta\beta_y \delta\beta_z t_{2,1} + (\delta\beta_z)^2 t_{1,2},$$

$$cz_s = t_{0,1} - \delta\beta_y t_{1,1} - \delta\beta_z t_{0,2} + (\delta\beta_y)^2 t_{2,1} + 2\delta\beta_y \delta\beta_z t_{1,1} + (\delta\beta_z)^2 t_{0,3},$$

and the conditions to be satisfied follow on solving the last three equations for $\delta\beta_y$ and $\delta\beta_z$. If we put

$$\frac{\theta}{y_s} = \frac{\phi}{z_s} = \frac{1}{x_s + y_s\beta_y + z_s\beta_z}$$

we have to a first approximation

$$\frac{\delta\beta_y}{\begin{vmatrix} 1 & t_{00} & \phi t_{00} \\ \theta & t_{10} & t_{11} \\ \phi & t_{01} & t_{02} \end{vmatrix}} = \frac{\delta\beta_z}{\begin{vmatrix} 1 & \theta t_{00} & t_{00} \\ \theta & t_{20} & t_{10} \\ \phi & t_{11} & t_{01} \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & \theta t_{00} & \phi t_{00} \\ \theta & t_{20} & t_{11} \\ \phi & t_{11} & t_{02} \end{vmatrix}}$$

Most often the corrections determined from these equations will be small enough for further work to be unnecessary. If desired, a closer approximation may be obtained by writing

$$\begin{aligned}
&\theta\, t_{00}\,(1-\delta\beta_y\theta-\delta\beta_z\phi),\\
&\phi\, t_{00}\,(1-\delta\beta_y\theta-\delta\beta_z\phi),\\
&t_{20}-\delta\beta_y t_{30}-\delta\beta_z t_{21},\\
&t_{11}-\delta\beta_y t_{21}-\delta\beta_z t_{12},\\
&t_{02}-\delta\beta_y t_{12}-\delta\beta_z t_{03}
\end{aligned}$$

in these equations instead of θt_{00}, ϕt_{00}, t_{20}, t_{11}, t_{02} respectively. In these five expressions $\delta\beta_y$ and $\delta\beta_z$ will be given the approximate values first found. If the first and second approximations differ appreciably the t's should be recomputed using corrected values of β_y and β_z.

THE STATIONARY VALUE OF AXIALLY SYMMETRIC FUNCTIONS. PART II

By T. SMITH, M.A., F.Inst.P., F.R.S.

(Correction and Addition.)

MS. received, 28th June, 1932.

In formula (40) on p. 295, Vol. 28 of these *Transactions*, the coefficients in the penultimate bracket should sum to the value 11340, the inverse of which occurs as a general factor. As printed the sum has not this value, but this will be secured if the term $+ 2003516D^4ED^2$ has its sign changed. The error was noted in applying the formula. It has not been possible to check the correction against the original manuscript, but there is little doubt that this emendation will make the expression correct.

It has also been noticed that in the last bracket a term in DED^2E^2 is present, although ED^2E^2 does not occur in the previous bracket. The formula has not therefore been simplified entirely in accordance with the original intention. Time has not been available for the somewhat heavy work of revising the whole expression. It may however be of value to record the expression reached in the penultimate stage before any "arbitrary" eliminations are made. The first five lines of (40) as printed are free from these arbitrary selections. The remaining terms are

$$\begin{aligned}
&\tfrac{1}{2520}(105E^3D + 26865E^2DE - 30720EDE^2 - 11270DE^3 - 22374E^2D^3 \\
&\quad + 33590EDED^2 - 5580ED^3E + 6454DE^2D^2 + 3245DED^2E \\
&\quad + 70353D^2EDE - 84856D^3E^2 + 4ED^5 + 8DED^4 - 66492D^2ED^3 \\
&\quad + 99772D^3ED^2 - 16592D^5E + 8D^7)\,u \\
&+ \tfrac{1}{11340}(93710E^4 - 43443E^3D^2 - 21715E^2D^2E - 547916EDEDE + 655142ED^2E^2 \\
&\quad - 373492DE^2DE + 440088DEDE^2 + 295724D^2E^3 + 14E^2D^4 + 508064EDED^3 \\
&\quad - 762074ED^2ED^2 + 127034ED^4E + 319952DE^2D^3 - 479860DEDED^2 \\
&\quad - 160178D^2E^2D^2 + 80020DED^3E - 80060D^2ED^2E - 1682956D^3EDE \\
&\quad + 2043886D^4E^2 + 4ED^6 + 8DED^5 + 12D^2ED^4 + 1602848D^3ED^3 \\
&\quad - 2404228D^4ED^2 + 400748D^6E + 8D^8)\,u \\
&+ \tfrac{1}{226800}(1890E^4D + 13898582E^3DE - 16123845E^2DE^2 - 9766889EDE^3 \\
&\quad - 7173898DE^4 - 11726324E^3D^3 + 17590416E^2DED^2 - 2931386E^2D^3E \\
&\quad + 5190160EDE^2D^2 + 2595320EDED^2E + 56044990ED^2EDE - 67876774ED^3E^2 \\
&\quad + 3517716DE^3D^2 + 1759034DE^2D^2E + 433188810DEDEDE \\
&\quad - 51868869DED^2E^2 + 51058828D^2E^2DE - 59948811D^2EDE^2 \\
&\quad - 36792054D^3E^3 + 56E^2D^5 + 32EDED^4 - 53140024ED^2ED^3 \\
&\quad + 79710144ED^3ED^2 - 13284924ED^5E + 128DE^2D^4 - 40281256DEDED^3 \\
&\quad + 60422100DED^2ED^2 - 10070150DED^4E - 43599904D^2E^2D^3 \\
&\quad + 65400372D^2EDED^2 - 10899762D^2ED^3E + 20180656D^3E^2D^2 \\
&\quad + 10090504D^3ED^2E + 210738180D^4EDE - 256026424D^5E + 16ED^7 \\
&\quad + 32DED^6 + 48D^2ED^5 + 64D^3ED^4 - 200878416D^4ED^3 + 301317840D^5ED^2 \\
&\quad - 50219440D^7E + 32D^9)\,u
\end{aligned}$$

and from these brackets 2, 5, and 14 terms respectively may be eliminated by means of the identities (37). The values already given in (40) are not necessarily those with the smallest numerical coefficients. For example, an alternative form for the first group of terms into which the arbitrary factors enter is

$$\tfrac{1}{7620}(105E^3D - 630E^2DE + 51765EDE^2 - 66260DE^3 - 49840EDED^2 \\ - 32130ED^3E + 67510DE^2D^2 + 83840DED^2E - 68548D^3E^2 + 32620ED^5 \\ - 81532DED^4 - 1260D^2ED^3 + 83464D^3ED^2 - 16592D^5E + 8D^7)\,u.$$

VISION

A DISCUSSION AT A JOINT MEETING OF THE PHYSICAL AND OPTICAL SOCIETIES, 3RD JUNE, 1932.

The following papers were presented:

INTRODUCTORY REMARKS. By J. Guild.

THE TRICHROMATIC THEORY AND ITS EXPLANATORY POWER. By F. Allen.

THE FUNDAMENTAL COLOURS AND THE COLOUR TRIANGLE. By J. Drever.

THE INTERPRETATION OF QUANTITATIVE DATA IN VISUAL PROBLEMS. By J. Guild.

COLOUR VISION BY MODIFIED WHITE LIGHT. By H. Hartridge.

A QUANTITATIVE FORMULATION OF COLOUR VISION. By S. Hecht.

SOME RECENT EXPERIMENTS ON THE SENSITIVENESS OF THE EYE TO DIFFERENCES IN THE SATURATION OF COLOURS. By L. C. Martin, F. L. Warburton, and W. J. Morgan.

THE ESSENCE AND PRESENT POSITION OF THE TRICHROMATIC THEORY. By W. Peddie.

THE MEASURABILITY OF SENSATIONS OF HUE BRIGHTNESS OR SATURATION. By L. F. Richardson.

THE SIGNIFICANCE OF COLOUR FATIGUE MEASUREMENTS. By W. D. Wright.

UNTERSUCHUNGEN ZUR DUNKELADAPTATION DES MENSCHLICHEN AUGES. By A. Brückner.

NEW OBSERVATIONS ON THE WEBER-FECHNER LAW. By R. A. Houstoun.

A NEW THEORY OF BRIGHTNESS CONSTANCY; A CONTRIBUTION TO A GENERAL THEORY OF VISION. By K. Koffka.

THE SENSATION OF LIGHT AS A PHOTO-CHEMICAL PROCESS. By D. Roaf.

EQUIVALENT ADAPTATION LEVELS IN LOCALISED RETINAL AREAS. By W. S. Stiles and B. H. Crawford.

THE COLOUR TRIANGLE AND COLOUR DISCRIMINATION. By T. Smith.

RETINAL ACTION TIME. By H. Banister.

A NOTE ON THE VISUAL PERCEPTION OF DEPTH. By F. C. Bartlett.

A STUDY OF THE REFRACTIVE CONDITIONS FOR THE PERIPHERAL FIELD OF VISION. By C. E. Ferree and G. Rand.

THE PHYSIOLOGICAL SIGNIFICANCE OF THE RETINAL SYNAPSES. By R. A. Granit.

SOME BIOLOGICAL ASPECTS OF VISUAL MEASUREMENTS. By J. Parsons.

LES LOIS DU TEMPS DU CHROMA. By H. Pieron.

DIE LEHRE VON DER AKKOMMODATION IM LICHTE DER NEUEREN FORSCHUNG. By A. von Pflugk.

VISUAL ACUITY AND RETINAL STRUCTURE. By R. S. Creed.

THE MECHANISM OF ACCOMMODATION AND THE RECESSION OF THE NEAR POINT. By E. F. Fincham.

VISUAL ACUITY AND THE RESOLVING POWER OF THE EYE. By H. Hartridge.

SOME EXPERIMENTAL OBSERVATIONS ON THE PROPERTIES OF THE RECEIVING ORGANS IN THE RETINA. By H. E. Roaf.

A SUBJECTIVE EXAMINATION OF EYE ASTIGMATISM. By F. Roessler.

REVIEWS

Experimental Studies in Diffraction. By Fredk. W. Shurlock. *Journ. Roy. Micr. Soc.* **51** 1931 (24–31, 127–35, 272–81, 408–13); 4 Figs. and 10 Plates. Price 5*s*. net.

A certain number of illustrations of diffraction effects—mostly diagrammatic—are to be found in numberless textbooks and other papers on optics. Probably no textbook dealing with microscope optics has been published for half a century which does not reproduce a few illustrations of the classical Abbe diffraction experiments with gratings, which led Abbe to his theory of microscope image formation, but it is rare to find more than one or two isolated reproductions of actual photographs in such publications.

In the experimental studies by the late Mr F. W. Shurlock the illustrations, nearly 180 in number, are reproduced from the actual photographs, excellently taken with great care, and in a subject of this nature the value of being able to study photographs in place of diagrams is self-apparent. Although the photographs are reproduced in half-tone, the fine detail is brought out remarkably well.

Mr Shurlock's experiments, intended to demonstrate the fundamental facts of diffraction and to show how they bear upon the theory of the microscope image, are divided into 4 series:

(1) Experiments illustrating the wave theory, comprising experiments with slits, pinholes, and apertures of various shapes and sizes, gratings, zone plates, and other diffraction effects with bi-prisms, Fresnel mirrors, etc.

(2) Experiments intended to illustrate the formation of images.

(3) Abbe's diffraction experiments with simple and crossed gratings.

(4) Diffraction from geometrical patterns.

Concise details are given how each photograph was obtained, so that the experiments can easily be repeated, and the descriptive comments indicate the nature of the conclusions to be drawn. With the exception of the 3rd series of experiments, which have been carried out with the microscope in the usual way, they have mostly been done with very simple large-scale apparatus with relatively coarse objects.

It is not always possible to accept the author's inferences, but the whole paper must be regarded in the light of the period in which it was written, which is shown by the following extract from his introductory remarks: "Abbe's diffraction experiments appear to have attracted little attention from physicists. They have, however, been the source of much controversy amongst microscopists, in the course of which it has sometimes been suggested that the behaviour of light in the microscope differs in some way from its behaviour in other circumstances. It therefore appeared worth while to investigate these experiments and to endeavour to repeat them on a larger scale apart from the microscope." Abbe's diffraction theory was indeed given to the world over 50 years ago as if its laws applied only to objects of those minute dimensions with which the microscope deals. His views at that time appeared in all textbooks, were constantly repeated, and in spite of his recantation of this misconception at a later date the impression became prevalent among microscopists that the diffraction phenomena appertained only to very fine gratings and other microscopic preparations, such as Abbe had originally worked with. Indeed almost to the present time his theory has been misunderstood by those with an insufficient acquaintance with physical optics. It was under those conditions then that Mr Shurlock's paper was written.

Neither Abbe nor Mr Shurlock, so far as can be gathered from their writings, appears to have been aware of the studies in diffraction by Schwerd, published as long ago as 1835*. These experiments were made with coarse gratings and apertures of all descriptions, comparatively large lenses of long focus being employed. Schwerd's great work is also profusely illustrated with diagrams, partly hand-coloured. Photography was of course only emerging from its infancy at that period. Many of Mr Shurlock's experiments are on somewhat similar lines to those of Schwerd, and his extensive series of photographs form an interesting complement to the analytical diagrams of Schwerd.

There is little doubt that the Royal Microscopical Society has rendered a general service in publishing the late Mr Shurlock's photographs with the text in its entirety. Gathered together in small compass the comprehensive series of photographs of diffraction phenomena is probably unique, and the paper should appeal to many besides those specially interested in the optics of the microscope image.

J. R.

Traité de Télémétrie. Par P. Mazuir. Pp. xix + 329; 261 Figs. and 15 Plates. (Paris: Revue d'Optique théorique et instrumentale, 1931.) Price 54 fr.

This is a comprehensive textbook on rangefinding, a subject of evergrowing interest and importance, and differs from other books available to the general reader, which in most cases deal only with the self-contained base, monostatic, or, as it is popularly called, "one man" rangefinder. In this book the reader will find a reasoned account of the principles and construction of practically all types of rangefinder and heightfinder.

The author is to be commended for the excellent way in which he has arranged this work. The volume opens with a simple introduction to the optical principles underlying the construction of rangefinders and their accessories, which is followed by a short section entitled physiological optics. The latter, which the author states is based on the work of Tscherning, contains a brief summary of the principal properties of the eye, a short chapter on monocular vision, and a more extended account of binocular vision.

The larger part of the book, some 250 pages, deals with the main subject, and is conveniently divided into parts each concerned with one main class of instrument. Each part commences with a theoretical chapter on the main principles of the class of instrument under review, followed by chapters describing the principles and construction of typical instruments of this class. This arrangement is logical and of great convenience to the reader.

Monostatic rangefinders are considered in much greater detail than those under other categories, principally because the former are better known, being the products of manufacturers *ab initio*, whereas most of the others have their origin in the Services, and in many cases particulars cannot be made available to the public. Moreover, the monostatic instrument has several uses in every-day life, and many of the principles involved in it are capable of being used with advantage in the design of instruments for other sciences. The author gives a full account of the use of such rangefinders, and a full discussion of the various methods employed in adjusting them. Both stereoscopic and coincidence instruments are treated.

The most recent application of the monostatic instrument is to heightfinding and this application more than any other has given rise to many new designs. The early monostatic heightfinder or altimeter was simply a rangefinder combined with a device for measuring the altitude of the target. The height was determined by calculation by slide rule or otherwise. Now instruments are made which are self-contained, measure all the quantities required, and perform the calculation of height automatically. The special interest of these instruments lies in the optical and mechanical means employed for performing these calculations. Captain Mazuir describes several of these instruments, and concludes his book by adding as an appendix a most valuable article on these devices by M. de Gramont which appeared in the *Revue d'Optique*. The study of the principles employed in these instruments is of value, not only to specialists in the use or manufacture of instruments for the Services, but to the general user or designer of measuring instruments. Critical accounts of such devices of British, German, and French origin will be found, which are worthy of study. There are also chapters on the selection and training of rangetakers.

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* *Die Beugungserscheinungen aus den fundamentalen Gesetzen der Undulationstheorie analytisch entwickelt und in Bildern dargestellt*. Von F. M. Schwerd (Mannheim: Schwan und Goetz, 1835).

The author is very fair in the criticisms he makes of the instruments and methods he has described, and has spared no pains in searching for material for this treatise. Very few instruments of which particulars are available and which are worthy of inclusion in such a book have been omitted. Professor Charles Fabry has supplied an excellent foreword to the book and most readers will be in cordial agreement with his closing sentence, "Ce sont surtout des problèmes posés dans les armées de terre et de mer qui ont conduit au perfectionnement des méthodes télémétriques; mais les appareils sont maintenant arrivés à un degré de perfection et à une facilité de maniement qui leur ouvrent beaucoup d'autres applications."

The book is well illustrated with clear diagrams, and the typography is of the high quality usually associated with the publications of the *Revue d'Optique*.

W. B. C.

Photogrammetry. Collected Lectures and Essays. Edited by O. VON GRUBER. Translated from the German Original by G. T. McCaw, M.A., and F. A. Cazalet. Pp. xii + 454; 353 Figs. and Inset Map. (London: Chapman & Hall, Ltd., 1932.) Price 30*s*. net.

The great interest taken in photogrammetry on the Continent may be realised from the large number of instruments designed and patented for facilitating one or more of the processes involved in constructing maps from photographs, and by the congresses and courses which have been held to discuss and elucidate the problems presented by this method of surveying. Though the subject has not been neglected in this country, it can hardly be said to have aroused the interest it deserves. The prompt translation of this account of the lectures forming the Sixth Vacation Course on Photogrammetry held at Jena in 1929, makes it possible for all English readers to appreciate the points of view and the stage recently reached by German and other continental workers.

On the whole the translators have done their work well, though in a few places the meaning tends to be obscure and has to be reconstructed from the context and illustrations; at times the construction of the original text is followed too closely to be pleasing, and at least one strange abbreviation occurs—no. Ap. for N.A. However these are very minor faults, and the book should serve its purpose well.

There are in all seven contributors, but in effect the volume consists almost entirely of lectures by members of the firm of Carl Zeiss. Dr R. Finsterwalder gives an entertaining account of experiences on the Alai-Pamir Expedition of 1928, with illustrations of the work that can be carried out in difficult country with a light photo-theodolite. The other contribution from a non-member of the firm is the concluding chapter by Dr L. Fritz on the efficacy of photogrammetry. In the course of this lecture the cost and time of Air Survey is compared with that of Planetabling to the advantage of the former, but it may be questioned whether this particular comparison is in fact a representative one. We may note that only the useful time of flying is included in the tables.

Four chapters are devoted to the materials and apparatus for photography. Dr K.

Gundlach contributes a short chapter on sensitive materials, which is illustrated by photographs showing variations in the rendering of detail by various types of film. The following chapter, by Dr W. Merté, describes Zeiss lenses suitable for photogrammetry. The next two chapters are by Dr H. Küppenbender. The second of these describes various forms of the Zeiss Air Camera, and the first is devoted to a very interesting and comprehensive examination of the properties of different forms of photographic shutter. This chapter should be of value to all interested in high-speed photography. The remaining chapters deal with the geometrical principles of photogrammetry and with the development of special instruments for the rapid construction of maps from photographic records. With the exception of a long chapter by Dr W. Sander on the development of photogrammetry in the light of invention, with special reference to plotting from two photographs, all the rest of the book (six chapters) is written by Dr O. von Gruber. The geometry of photogrammetry is developed at considerable length, but it is not clear that in practice much of this theory is utilised. The treatment however is clear, and serves as an adequate introduction to the subject. The parts of the greatest interest however are the later chapters and that of Dr Sander, where the development of the elaborate instruments that have been produced in the last thirty years on the Continent is described. These accounts are of the greater interest as they are written by men who have witnessed and contributed to the growth of many of these instruments from simple beginnings and have told how one idea was added to another or one rejected in favour of another. The story should prove of interest to a much wider circle of readers than those concerned with the technical aspects of this form of surveying. Examples of work carried out with various instruments are reproduced, and, as would be anticipated, the book is very well and fully illustrated, and should be useful for reference. As a preface there is a short appreciative notice, with a portrait, of the late Dr Carl Pulfrich, by whom these courses on photogrammetry were founded.

T. S.

Professional Papers of the Air Survey Committee.

No. 4. The Stereoscopic Examination of Air Photographs. Pp. 84; 47 Figs. and two photographs in pocket. 1927. Price 3*s*. 6*d*. net.

No. 5. Calibration of Surveying Cameras. Pp. 81; 35 Figs. 1929. Price 2*s*. 6*d*. net.

No. 6. Extensions of the "Arundel" method. Pp. 115; 40 Figs. and 4 Plates. 1929. Price 4*s*. 0*d*. net.

By Capt. M. HOTINE, R.E. (London: H.M. Stationery Office.)

These monographs describe aspects of work carried out at the War Office in developing methods utilised in this country in surveying from the air. The tendency has been to simplify the procedure as far as possible by the selection of special conditions and the use of comparatively inexpensive apparatus. The first of these essays deals with the principles of stereoscopic vision, stereoscopic reconstruction and relief, and types of floating mark, leading up to the description of the reflecting stereoscope now used extensively in contouring and in other work on strip photographs. The second is mainly concerned with various methods, suitable for use in the field, of calibrating surveying cameras. The effect of the pressure plate on the distortion is described. Laboratory methods are also outlined, but of these some have apparently not been tried. No indication is given of the relative merits of the methods mentioned. The third paper deals with the extension of the "Arundel" method of topographical survey to country where the variations in height of the ground form a larger fraction of the flying height than was contemplated in the earlier trials of this method. The principle used is essentially that of radial plotting, and the distinctive feature of the present work is the necessity for correcting the effects of tilt of the plate to a higher order than is called for when the ground is less irregular. Advice on many points of practical importance, based on experience gained in official trials, is given, and a routine for the computations involved is suggested. The description would have gained in interest and clarity had the essential problems been discussed apart from particular methods of computation. The computational problems themselves could also with advantage have been stated separately in a general form, and then followed by an account of the method of solution preferred by the author. The equipment and the experience of the staff concerned are factors of much importance in organised computational work, and it is rarely possible to claim that a particular routine is the most advantageous in all circumstances. In any event descriptions of numerical computations make dull reading.

The development described in this paper is however of decided importance, and is sure to be utilised extensively in the future when maps of fair accuracy have to be produced rapidly for engineering or military purposes.

T. S.

Vision and Colour Vision. By R. A. HOUSTOUN, M.A., D.Sc. Pp. vii + 238; 102 Figs. (London: Longmans, Green & Co., 1932.) Price 15*s*. net.

During the past twenty years Dr R. A. Houstoun and his pupils have carried out a series of important researches in colour vision and the general nature of the visual process. The present book brings the results of this work into their relation with our general experimental knowledge of vision.

The various "classical" experiments such as those of Maxwell, König and Brodhun, and others are described in a simple and attractive manner; the main feature of the experiments and their results are well brought out, and we should be in Dr Houstoun's debt for these parts of the book if for no other reasons. He does not devote over-much space to his own experiments and theories, which might indeed have been more fully discussed, in several instances, with advantage. The manner in which he illustrates the yellow-spot effect and the Purkinje phenomenon with the results of simple experiments is much better than the indirect and involved descriptions usual in text-books.

This is not the place for controversy regarding the theoretical views put forward, but when we find that the "Young-Helmholtz" theory is held to "violate commonsense," and to be mathematically untenable, it will be realised that the argument lacks nothing in vigour. The nature of the Young-Helmholtz theory is clearly enough stated (in section 91) but the reader may be forgiven for asking whether perhaps the mathematical difficulties do not arise in trying to force it into an inevitable physiological framework.

Dr Houstoun's own theories are very ingenious, and certainly merit careful consideration, especially with a view to establishing the likelihood of some yet unknown effect which might be verified by experiment. Has *any* major visual phenomenon yet been predicted by any theory?

Apart from theory it seems to the present writer that the most permanent value of the original work lies in the extensive surveys of colour vision which have been carried out in Glasgow. These have considerably advanced our knowledge of the incidence of colour blindness, although the very originality of Dr Houstoun's methods makes it somewhat difficult to compare his results with those of other workers.

It would appear that a very considerable revival of interest in the whole subject is taking place and the book as a whole may be heartily commended, if not for removing all difficulties, at any rate for giving a refreshing and stimulating review of one of the most fascinating puzzles that the human mind has yet to solve.

L. C. M.

Introduction to Physical Optics. By Prof. JOHN KELLOCK ROBERTSON, F.R.S.C. Pp. vii + 422; 223 Figs. and 6 Plates. (London: Chapman and Hall, Ltd., 1931.) Price 20*s.* net.

Although numerous books on physical optics have been published, very few, if any, of them provide a comprehensive introduction to the subject. The author of the present book is therefore right when he states in the preface that "no apology is needed for placing this one before the public." Prof. Robertson's general treatment of the various branches of the subject is well planned and, although in a number of instances the phraseology is not all that could be desired, the student should find no difficulty in understanding every chapter. The theoretical portions are made very clear, the mathematics involved being of a simple nature. The chief merit of the book is, however, the way in which the author deals with the apparent contradictions between the classical wave theory and the more modern theoretical developments. The main facts are very plainly, though of necessity concisely, stated so that the reader is not left in rather a bewildered state which stifles further interest. On the contrary the average student, after mastering Prof. Robertson's book, will be sure to want to know more about the recent developments of relativity, wave mechanics, and kindred subjects. This whetting of the appetite for further knowledge is, or at least should be, one of the chief aims of the writer of an introductory treatise. The book is well printed and the diagrams are good, though the lettering in some of them is not too clear.

The Practice of Spectrum Analysis with Hilger Instruments. Compiled by F. TWYMAN, F.Inst.P., F.R.S. Fifth edition. Pp. 53 + 7; 6 Figs. (London: Adam Hilger, Ltd., 1931.) Price 3*s.* 6*d.* net.

This booklet can best be described as a condensed precis of the work, carried out up to the present, on the applications of spectroscopic methods to problems in mineralogical, metallurgical and general chemical analysis. It is a veritable mine of information on the possibilities of both qualitative and quantitative analysis in these fields and it is of interest to note that a not inconsiderable part of this knowledge has been contributed by the research laboratories of the publishers.

Full details of the various examples that are quoted would form an encyclopaedic volume; the real usefulness of this little handbook lies in the fact that the reader can quickly find what has been done in reference to his own particular problem and where he can find a full account of it. The beginner is shown how to take spectrograms, measure wave-lengths and identify the elements, while the experienced worker will appreciate the valuable classified bibliography, the compilation of which must have entailed a considerable amount of bibliographical research.

The obvious popularity and success of this booklet may induce the publishers to complete the subject by preparing a companion volume on gases and particularly on molecular spectra. The line spectra will merely tell the nature and amount of the metallic base that is present; it would clearly be of much greater interest to know whether the sample contained the oxide, nitrate or other salt. The enormous progress, made during the last few years both in theoretical and practical work on band spectra, makes this complementary method of spectroscopic analysis of almost equal importance.

W. E. W.

The British Association for the Advancement of Science: A Retrospect 1831–1931. Centenary (Second) Edition. By O. J. R. Howarth, O.B.E., M.A. Pp. vii + 330; 1 Fig. and 19 Plates. (London: The British Association, 1931.) Price 3*s*. 6*d*. net.

This volume was prepared for the centenary meeting of the Association in London last year and is a revision of the first edition published ten years ago and brought up to date. The history and development of the Association are very attractively summarised in a very readable form. The subject matter is essentially one for a much bigger volume or volumes but enough is given to show the important part which the Association has played in the discussion and encouragement of science in Great Britain. The book is illustrated by a number of photographs of past presidents closely identified with the history of the Association. Two appendices give particulars of grants made by the Association for actual research since its formation and biographical details of past presidents.

H. B.

London and the Advancement of Science. By various authors (edited by O. J. R. Howarth). Pp. v + 321. (London: The British Association, 1931.) Price 3*s*. 6*d*. net.

The volume consists of nine essays, all having some connection with the main theme of London and Science. Dr Ferguson, whose original suggestion led to the preparation of the book, commences with an Introductory Survey in which he very charmingly discourses on a number of representative men of science who have been closely associated with London. The second essay deals with the Learned Societies and is followed by one on Education in London. The London University problem, it appears, is one of long standing and plans for an academy in London in which science should be taught date back to the time of Sir Francis Bacon and Sir Humphrey Gilbert. One misses any reference in this chapter or in Dr Ferguson's to Stephen Grey of Charterhouse and his electrification of his pupils.

Other essays deal with the Government and Scientific Research by Sir Frank Heath, the Royal Observatory by Sir Frank Dyson, the Museums of London by Dr F. Bather, and a brief History of the London Makers of Scientific Instruments by Mr R. S. Whipple. This most interesting essay, particularly to the readers of this journal as it deals mainly with the manufacturers of optical instruments, is disappointingly short. It is clear, however, that the subject of the London makers of instruments is one on which considerably more of interest can be written.

As the book was prepared for the occasion of the London meeting of the British Association, it is possible that it may share the fate of the semi-ephemeral literature associated with congresses, exhibitions, etc. If that is so it is a pity, as this book represents a relatively permanent short study of a great subject.

H. B.

Annual Report of the Smithsonian Institution, 1930. Pp. xii + 650; 56 Figs. and 191 Plates. (Washington: U.S. Government Printing Office, 1931.) Price $2.00.

The *Annual Report of the Smithsonian Institution* furnishes its usual *olla podrida* of interesting and varied fare. The Secretary's report occupies some 160 pages and gives, amid a mass of essential (and sometimes dull) financial and routine detail, occasional glimpses of activities concerning which outside observers would welcome fuller information. For example, in one department we find extracts submitted from remarks made by

Dr E. C. Richardson on the advisability of reviving the International Catalogue of Scientific Literature. Even in the present disturbed economic state of the world (possibly because of that state) a convincing case may be presented for the furbishing of three of the essential tools of the man of science—a catalogue of literature, abstracts, and handbooks. Of these Dr Richardson urges that a good case may be made out for the international catalogue as the one essential and indispensable tool. If that be so, obviously it is important that the mechanism of the international catalogue be not allowed to rust away into nothingness. The task of building up *ab initio* such an organisation might well prove beyond the powers of the mad world in which we live, and the Smithsonian Institution is performing a public service of the first magnitude in urging the revival of this most important work.

The 500 pages of the general appendix comprise a collection of some 29 essays covering a remarkably wide range of the physical and natural sciences, each essay written by an authority on the subject, and most of them models of the art of *haute vulgarisation scientifique*. Essays on the autogiro, gliding in Germany, modern concepts in physics, weather and glaciation, waves and corpuscles, man and insects, the mechanism of organic evolution, the Holland vehicular tunnel, represent but a small selection from the topics, freshly and authoritatively treated, whose only fault is that, in the nature of the case, they appear in the report some two years after their first publication; and, in the realm of physical science at least, two years is a long time.

That Dr Abbot is responsible for the report is sufficient guarantee of its accuracy, balance, and width of view.

A. F.

Photograms of the Year 1931*: The Annual Review for* 1932 *of the World's Pictorial Photographic Work*. Edited by F. J. Mortimer, F.R.P.S. Pp. 24 + 28; 65 Plates. (London: Iliffe & Sons, Ltd., 1931.) Price (paper covers) 5*s*. net., (cloth boards) 7*s*. 6*d*. net.

In this publication "an endeavour is made annually to present to the picture-loving public a volume of reproductions indicating the most recent developments of pictorial photography as provided by some of the best workers of the world." The amateur photographer who aspires to a high standard in his work will benefit greatly from a detailed study, not only of the many beautiful photographs reproduced in this volume, but also of the comments provided by C. J. Symes, which occupy about eight pages of text. In addition the volume contains a review of the year's work by the editor, short contributions from Australia, America, Canada, Germany, France, Belgium, New Zealand, Czecho-Slovakia, Poland, Egypt, Japan, and Spain, and a complete directory of all the British Photographic Societies, Camera Clubs, and Amateur Cinematograph Societies.

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PROCEEDINGS
AT THE
MEETINGS OF THE OPTICAL SOCIETY
1931–32

15th October, 1931

Meeting held at the Imperial College of Science.

Prof. A. F. C. Pollard, Vice-President, in the Chair.

An appreciation of the late Prof. A. Barr, a Past President of the Society, was delivered by Mr W. B. Coutts.

The following papers were read:

"The Validity of the Schwarzschild Relation as Applied to the Use of the Logarithmic Sector." By F. Twyman, F.Inst.P., F.R.S., and A. Harvey, Ph.D., B.Sc.

"The 'Spekker' Photometer for Ultra-violet Spectrophotometry." By F. Twyman, F.Inst.P., F.R.S. (Read by Dr A. Harvey.)

By the courtesy of the Director of the Science Museum a visit was paid to a special exhibit of optical instruments in the Museum, Mr D. Baxandall acting as guide.

10th December, 1931

Meeting held at the Imperial College of Science.

Prof. A. O. Rankine, President, in the Chair.

A letter of thanks to the President from General Smuts, President of the British Association Centenary Meeting, for the help of the Society was read.

Elections (Membership): Mr G. E. Cox and Mr B. K. Johnson.

The following paper was read:

"A New Photo-electric Spectrophotometer." By F. Campbell Smith, B.Sc., M.R.C.S., and Ensor R. Holiday, B.M., M.R.C.S.

The following was demonstrated:

"Methods of Focussing Projectors." By J. M. Waldram, B.Sc. (Eng.).

The following were exhibited by Messrs W. Watson & Sons, Ltd.:

The "Service" Microscope.
The "H" Edinburgh Student's Microscope.
The Works Model of "Service" Microscope.
A Low-power Stereoscopic Dissecting Microscope.
A Modification of the Usual Form of Eye-piece Camera.

5th, 6th and 7th January, 1932

Twenty-second Annual Exhibition of Electrical, Optical and other Physical Apparatus held at the Imperial College of Science, jointly with the Physical Society.

The following discourses were given:

"Photocells; the Valves which Operate by Light." By C. C. PATERSON, O.B.E., M.I.E.E., F.Inst.P.

"Photographic Shutters and their Properties." By T. SMITH, M.A., F.Inst.P.

"Reminiscences." By Sir OLIVER J. LODGE, D.Sc., LL.D., F.Inst.P., F.R.S.

11th February, 1932

Meeting held at the Imperial College of Science.

Prof. A. O. RANKINE, President, in the Chair.

It was announced that the Council had appointed Dr H. R. LANG Assistant Secretary of the Society.

The following papers were read:

"A Magnification Method of Measuring Focal Length." By JOHN S. ANDERSON, M.A., Sc.D., F.Inst.P.

"Rapid Spectrophotometry with Bi-multiple Spectra and a New Type of Wedge Cell." By F. TWYMAN, F.Inst.P., F.R.S., L. J. SPENCER, and A. HARVEY, Ph.D., B.Sc.

The following was demonstrated:

"Reflection of Coloured Lights from Etched Polycrystalline Metallic Surfaces." By J. L. HAUGHTON, D.Sc., F.Inst.P.

10th March, 1932

Meeting held at the Imperial College of Science.

Prof. A. O. RANKINE, President, in the Chair.

Elections (Membership): Dr J. F. SIRKS.

(Student Membership): Mr E. G. KNOWLES and Mr G. R. WATSON.

Comdr. N. M. S. LANGLANDS and Mr W. D. WRIGHT were elected to act as Hon. Auditors of the Society's accounts for the year 1931.

The result of the referendum with respect to the proposed amalgamation of the Optical and Physical Societies was announced.

The following papers were read:

"Note on the Absorption of Light." By R. A. HOUSTOUN, M.A., D.Sc.

"Note on Refractive Indices." By Mrs E. GIFFORD.

The following were demonstrated:

"The Manufacture of a Microscope, with Cinematograph Illustrations." By W. E. WATSON BAKER.

"Measurement of Particle Size and Concentration in Opal Glasses." By C. DUNBAR, M.Sc.

14th April, 1932

Meeting held at the Imperial College of Science.

Prof. A. O. RANKINE, President, in the Chair.

Election (Student Membership): Mr K. ROWE.

The following papers were read:

"A Photometric Eyepiece for Visual Quantitative Spectrum Analysis." By F. TWYMAN, F.Inst.P., F.R.S. (Read by Dr A. Harvey.)

"The C.I.E. Colorimetric Standards and their Use." By T. SMITH, M.A., F.Inst.P., and J. GUILD, A.R.C.S., F.Inst.P., F.R.A.S.

12th May, 1932

Meeting held at the Imperial College of Science.

Dr JOHN S. ANDERSON, Vice-President, in the Chair.

Elections (Membership): Mr F. N. ARCHBOLD and Mr C. REYERSBACH.

The following papers were read:

"The *F*/1·8 Quartz Monochromator-spectrograph." By J. W. PERRY.

"The Transmission of Light in Diffusing Media." By R. F. HANSTOCK, B.Sc., A.Inst.P.

3rd June, 1932

Meeting held at the Imperial College of Science.

Prof. A. O. RANKINE, President, in the Chair.

A discussion on "Vision" was held, jointly with the Physical Society.

9th June, 1932

Annual General Meeting held at the Imperial College of Science.

Prof. A. O. RANKINE, President, in the Chair.

The Hon. Secretary presented the Annual Report* of the Council, which was approved.

The Hon. Treasurer presented his Report† and Balance Sheet‡, which were approved.

The Chairman read the list of persons nominated by the Council for office in the session 1932–33 and declared all these duly elected as no other nominations had been received:

President: Professor A. O. RANKINE.
Vice-Presidents: Dr J. S. ANDERSON, Mr E. F. FINCHAM,
Professor A. F. C. POLLARD, Mr R. S. WHIPPLE.
Hon. Treasurer: Major E. O. HENRICI.
Hon. Secretary (Business): Mr W. B. COUTTS.
Hon. Secretary (Papers): Mr E. T. HANSON.
Hon. Librarian and Editor of Transactions: Dr J. S. ANDERSON.
Assistant Secretary: Dr H. R. LANG.

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* See p. ix. † See p. xiii. ‡ See p. xiv.

Council:

Already serving:
Instructor-Captain T. Y. Baker.
Mr T. Chaundy.
Dr C. V. Drysdale.
Mr S. A. Emerson.
Mr H. H. Emsley.
Mr W. Gamble.
Mr J. Guild.
Dr W. M. Hampton.
Professor L. C. Martin.

Elected June, 1932:
Dr R. S. Clay.
Mr T. Smith.
Mr F. Twyman.

Votes of thanks to the retiring Officers and Members of Council, to the Hon. Auditors, to the Governors of the Imperial College, with Professor G. P. Thomson, for the use of the College premises for the meetings of the Society, and to Messrs Banfield, Tonks, and Shepherd for their assistance at the meetings of the Society, were unanimously passed.

9th June, 1932

Meeting held at the Imperial College of Science.

Prof. A. O. Rankine, President, in the Chair.

It was announced by the Chairman that this was the last ordinary meeting of the Society and that a special meeting would be held towards the end of September to ratify the amalgamation of the Optical and Physical Societies.

The following papers were read:

"A New Trichromatic Colorimeter." By R. A. Houstoun, M.A., D.Sc. (Read by Mr J. W. Perry.)

"The Parallel Plate Micrometer." By Instr.-Capt. T. Y. Baker, F.Inst.P., R.N. (Ret.).

"A Study of the Cyclo-rotational Powers of the Eyes." By J. Adamson, M.Sc., A.Inst.P.

"The Hiding Power of Diffusing Media." By T. Smith, M.A., F.Inst.P., F.R.S.

The following was demonstrated:

"An Apparatus for Investigating the Path of Light through Fluted Glass Reflectors, etc." By the Research Staff of the General Electric Co., Ltd.

THE OPTICAL SOCIETY

REPORT OF THE COUNCIL TO THE ANNUAL GENERAL MEETING OF MEMBERS, 9TH JUNE, 1932

The Council of the Optical Society presents the following Report for the year ended 31st December, 1931.

During the year the Annual General Meeting and eight Ordinary Meetings of the Society were held at the Imperial College of Science and Technology by permission of the Governors and Professor Thomson, to whom the thanks and appreciation of the Society are due.

The average attendance at the meetings was 34. During the year fourteen papers were communicated and nine demonstrations were given at the Ordinary Meetings.

At the May Meeting, by invitation of the Council, Professor G. P. Thomson, F.R.S., delivered a lecture on "Electron Diffraction," with experimental demonstrations.

The Thomas Young Oration was delivered by Sir John H. Parsons, D.Sc., F.R.C.S., F.R.S., on 11th June. The subject of the Oration was "Young's Theory of Colour Vision (1801–1931)."

The Twenty-first Annual Exhibition of Optical, Electrical and other Physical Apparatus, under the auspices of the Physical and Optical Societies, was held at the Imperial College of Science and Technology, on the 6th, 7th, and 8th January, and surpassed that of 1930 with respect to the number of exhibitors and visitors.

The Twenty-first Exhibition was specially marked by an official opening ceremony performed by Sir Arthur Eddington, F.R.S., President of the Physical Society. The competition for apprentices and learners employed by exhibiting firms was held for the second time. The examples of craftsmanship and draughtsmanship presented by the entrants again showed individual skill of a high order. The thanks of the Society are due to Dr C. V. Drysdale, Professor A. F. C. Pollard, the late Sir Thomas Stanton and Mr William Taylor, who kindly undertook the laborious task of acting as judges of these exhibits.

During the Exhibition, discourses were delivered by Mr E. Lancaster Jones, B.A., on "Searching for Minerals with Scientific Instruments," and by Professor Sir Gilbert Walker, F.R.S., on "The Physics of Sport."

As in former years, members of the Society have been invited to attend the lectures on "Physics in Industry" by the Board of the Institute of Physics, and the meetings of the Illuminating Engineering Society.

The *Transactions* have been maintained at their customary high standard. The Council regrets that serious delay occurred in the publication of the early numbers of Vol. XXXII. Steps were taken to speed up publication of this volume, which has now been completed. Dr Hedges resigned the office of Editor on relinquishing the office of Secretary of the Institute of Physics and the *Transactions* are again being edited by Dr Anderson.

The Council records with deep regret the loss the Society has sustained by the death of three of its most distinguished Fellows, Professor Albert Abraham Michelsen (Honorary Fellow), the Hon. Sir Charles Parsons, O.M., K.C.B., D.Sc., LL.D., F.R.S., and Emeritus Professor Archibald Barr, D.Sc., LL.D., F.R.S., President of the Society 1923–5.

Professor A. O. Rankine, President, was the official delegate of the Society to the Centenary Meeting of the British Association for the Advancement of Science, and to the Faraday Celebrations held in London in September, 1931.

Dr Clay represented the Society on the National Committee of Physics. Mr Twyman continued to represent the Society on the Board of the Institute of Physics.

The President represented the Society on the Joint Committee for the Standardisation of Tests of Photo-Electric Cells.

The Society was invited to send representatives to a meeting called by the Royal Society in July, 1931, to consider the question of a suitable memorial to the late the Hon. Sir Charles Parsons. The Society was represented on this occasion by the President, Mr Coutts, and Mr Whipple.

It was announced in the last annual report that the Council had been somewhat concerned at the comparatively small number of papers presented to the Society, and the relatively poor attendance at meetings. During the year under review the Council has given earnest and continued attention to this matter, and has considered various means for increasing the value of the Society's activities. Towards the end of the year the Council came reluctantly to the conclusion that the Society could not properly fulfil its purpose under present-day conditions. Many societies deal with optical matters in relation to other sciences, in some cases in competition with one another. It was felt that the separate existence of the Optical Society was not in the best interests of present-day optical science, and that those interests would be furthered most by an amalgamation of the Optical Society with another society which dealt with optical science and its applications on the widest possible basis. The Council thereupon resolved to explore the possibility of amalgamation with the Physical Society, with a view primarily to furthering optical science.

MEMBERSHIP

During the year eight Members and one Student were elected, and one transfer from Studentship to Membership was made. One Fellow, five Members, and two Students resigned, and one Student was removed from the roll. One Honorary Fellow, two Fellows, and one Member died. The Membership of the Society thus shows a decrease of two, the total number at 31st December, 1931, being 302, made up as follows:

Ex-officio Fellow	1
Honorary Fellows	9
Fellows	130
Members	145
Corporate Members	9
Students	8
Total	302

LIBRARY

In addition to the regular journals the following publications have been received during 1931:

Modern Cosmologies. By Hector Macpherson, M.A., Ph.D.

Die optische Werkstatt: Handbuch der Arbeitsverfahren und Prüfmethoden für die Fertigung von Optik. Von Dr Waldemar Ewald unter Mitarbeit von Dr Hans Schulz und Dr Franz Weidert.

I. G. Farbenindustrie Aktiengesellschaft: Veröffentlichungen des wissenschaftlichen Zentral-Laboratoriums der photographischen Abteilung (*Agfa*), Band I.

Text-Book on Spherical Astronomy. By W. M. Smart, M.A., D.Sc.

Lens-Work for Amateurs. By Henry Orford. Revised by A. Lockett.

The Mathematical Papers of Sir William Rowan Hamilton, Vol. I. *Geometrical Optics.* Edited by A. W. Conway, D.Sc., F.R.S., M.R.I.A. and J. L. Synge, Sc.D., M.R.I.A.

Opticks or A Treatise of the Reflections, Refractions, Inflections and Colours of Light. By Sir Isaac Newton. Reprinted from the Fourth Edition.

Traité de Pyrométrie Optique. Par Gustave Ribaud.

Lezioni di Ottica Geometrica. Per Gino Giotti.

The Structure of Crystals. By Ralph W. G. Wyckoff. Second edition.

Quartz Resonators and Oscillators. By P. Vigoureux, M.Sc.

Strahlenoptik. Von Dr M. Herzberger.

Traité de Télémétrie. Par P. MAZUIR.

Dynamic Retinoscopy. By MARGARET DOBSON, M.D.

Newton: The Man. By Lieut.-Col. R. DE VILLAMIL, R.E. (Ret.).

Penrose's Annual, Vol. XXXIV. Edited by WILLIAM GAMBLE, F.R.P.S., F.O.S.

A Treatise on Photo-Elasticity. By Prof. E. G. COKER and Prof. L. N. G. FILON.

British Association for the Advancement of Science:
- *A Retrospect* 1831–1931. By O. J. R. HOWARTH, O.B.E., M.A.
- *London and the Advancement of Science.* By various authors.

The National Physical Laboratory:
- *Collected Researches*, Vol. XXII.
- *Report for the Year* 1930.

Bureau of Standards:
- Research Paper No. 262. *Prism Size and Orientation in Minimum-Deviation Refractometry.* By L. W. TILTON.
- Research Paper No. 318. *Measurement of Extreme Ultra-violet Solar Radiation by a Filter Method.* By W. W. COBLENTZ and R. STAIR.
- Research Paper No. 365. *A Correlated Color Temperature for Illuminants.* By RAYMOND DAVIS.
- Miscellaneous Publication No. 131. *Annual Report of the Director for the Fiscal Year ended June* 30, 1931.

Smithsonian Institution:
- *Annual Report*, 1930.

TREASURER'S REPORT FOR THE YEAR 1931

THE income for the year shows a diminution of £55. 9*s.* 7*d.* This is almost entirely due to a decrease in the receipts from the sales of publications, no runs of back numbers being sold during the year.

The expenses of issuing the *Transactions* are down for two reasons. The cost of Vol. XXXII will be below the normal, and the cost of Vol. XXXI, Part 5 was £100 less than the amount allowed in the accounts for 1930. This number had not been made up when the accounts were prepared. There were no expenses for reprints or special publications during the year. Other expenses were normal and the net result is an excess of income over expenditure of £368. 7*s.* 2*d.* for the year, bringing the accumulated funds up to £793. 14*s.* 6*d.*

During the year the investments were increased by the purchase of £200 War Loan.

The investments are, as usual, shown "at cost" in the Balance Sheet. The market value at 31st December, 1931 was £912, showing a depreciation of £54. Since then the price has again risen and this depreciation has been turned into a small appreciation.

As usual the assets shown in the Balance Sheet are all (with the possible exception of subscriptions due) liquid assets, and no credit has been taken for the value of the stock of publications, books or furniture, though this is considerable, and they are insured against loss by fire for the sum of £300.

E. O. HENRICI,
Hon. Treasurer.

10*th March*, 1932.

INCOME AND EXPENDITURE ACCOUNT FOR THE YEAR ENDED 31ST DECEMBER, 1931

1930 £ s. d.	*Dr.* EXPENDITURE	£ s. d.	£ s. d.
	To Printing and Stationery:		
539 4 11	Printing and Distributing *Transactions*	325 11 6	
29 11 10	Office Printing and Stationery	29 13 10	
		355 5 4	
	Less Excess reserves in Accounts for 1930	100 15 0	
			254 10 4
	Clerical Assistance:		
125 0 0	Institute of Physics (includes administration of Exhibition)	125 0 0	
50 10 6	Editorial Expenses	51 17 6	
15 9 9	Other Expenses	17 16 2	
			194 13 8
43 12 3	Postage and Receipt Stamps		37 13 3
16 4 9	Expenses at Meetings		13 11 11
8 4 9	Sundry Expenses		2 16 9
4 16 0	Advertising		2 9 0
162 7 9	*Reproducing rare parts of Transactions*		—
141 0 3	*Printing and Publishing "Photo-Electric Cells and Their Applications"*		—
14 14 0	*Printing List of Members*		—
	Balance, being excess of Income over Expenditure, transferred to Accumulated Fund		368 7 2
£1150 16 9			£874 2 1

1930 £ s. d.	INCOME *Cr.*	£ s. d.	£ s. d.
	By Subscriptions:		
263 9 11	Fellows	256 18 8	
152 14 11	Members	151 0 10	
28 7 0	Corporate Members	28 7 0	
3 3 0	Student Members	1 1 0	
447 14 10		437 7 6	
12 1 6	*Less* Rebate to Members of the Institute of Physics	13 15 9	
435 13 4			423 11 9
45 19 7	Advertisements in *Transactions*, *less* expenses		35 1 0
357 5 10	Sales of publications		304 5 1
37 15 2	Interest and Dividends		43 18 3
52 17 9	Physical Society—Exhibition Account		67 6 0
221 5 1	*Balance, being excess of Expenditure over Income, transferred to Accumulated Fund*		—
£1150 16 9			£874 2 1

OPTICAL CONVENTION, 1926. TRUST ACCOUNT

	£ s. d.
To Balance carried forward	173 18 1
	£173 18 1

	£ s. d.
By Balance brought forward from 1930	137 1 3
Sales of Publications	31 16 10
Dividends from Investment	5 0 0
	£173 18 1
By Balance brought forward	£173 18 1

BALANCE SHEET AT 31ST DECEMBER, 1931

LIABILITIES

	£	s.	d.	£	s.	d.
Sundry Creditors				607	14	10
Accumulated Funds:						
Balance at 31st December, 1930	425	7	4			
Add Balance of Income and Expenditure Account	368	7	2			
				793	14	6
Life Compositions Account:						
Balance at 31st December, 1930				94	10	0
Subscriptions received in advance				8	5	8
Optical Convention, 1926, Trust Account				173	18	1
				£1678	3	1

ASSETS

	£	s.	d.	£	s.	d.
Sundry Debtors				156	9	6
Subscriptions due				12	12	0
Investment:						
£950 5 % War Loan at cost				966	5	9
Cash at Bank: Deposit Account	400	0	0			
Current Account	130	16	3			
Cash in hand	11	19	7			
				542	15	10
				£1678	3	1

E. O. HENRICI,
Hon. Treasurer.

We have audited the above Balance Sheet and have obtained all the information and explanations we have required. We have verified the Bank Balances and the Investment. In our opinion such Balance Sheet is properly drawn up so as to exhibit a true and correct view of the state of the Society's affairs according to the best of our information and the explanations given to us and as shown by the Books of the Society.

KNOX, CROPPER & CO.
Chartered Accountants.

Spencer House,
South Place, E.C. 2.
14*th April*, 1932.

Examined and approved on behalf of the Society.

N. M. S. LANGLANDS,
W. D. WRIGHT,
Hon. Auditors.

6*th May*, 1932.